Paper 4 - Fundamentals of Business Mathematics and Statistics

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Time Allowed: 3 Hours **Full Marks: 100**

The figures in the margin on the right side indicate full marks.

This question paper has two sections.

Both the sections are to be answered subject to instructions given against each.

Section – A

I. (a) Choose the correct answer **(9 × 2 = 18)**

(1) The ratio of work done by (x + 2) men in (x – 2) days to that of (x – 1) men in (x + 1) days is 4:5, the value of x is

(a) ± 4, (b) 6, (c) 4, (d) 8

(2) The mean proportional between 2 and 8 is

(a) 4 (b) 16 (c) 3 (d) 1

(3) If one roots of the equation $x^2 - 3x + m = 0$ exceeds the other by 5 then the value of M is equal to _____

(a) -6 (b) -4 (c) 12 (d) 18

(4) If $^{n}p_3 = 120$ then n = ___

(a) 8 (b) 4 (c) 6 (d) None of these

(5) The 7th term of the Arithmetic Progression (AP) 7, 10, 13, 16, …….. is

(a) 28 (b) 22 (c) 25 (d) 20

(6) If $^{r}c_{12} = {}^{r}c_8$ find $^{22}c_r$

(a) 213 (b) 321 (c) 231 (d) None of these

(7) Some money is distributed between A and B in the ratio 2:3. If A receives ₹ 72, then B receives:

(a) ₹ 90 (b) ₹ 144 (c) ₹ 108 (d) None of these

(8) The value of logarithm of 1 to the base 10 is

(a) 2 (b) 10 (c) 0 (d) 1

(9) If $^{n}P_4 = 30 \times {}^{n}P_2$, then the value of n is

(a) 10 (b) 8 (c) 6 (d) 5

I. (b) State whether the following statements are true or false (6 × 1 = 6)

(1) The average of 50 numbers is 38. If two numbers, namely 45 and 55 are discarded, the average of the remaining numbers is 30. ( )

(2) The value of $^5C_2$ is equal to $^5C_3$. ( )

(3) The geometric mean of 3 and 1/3 is – 1 ( )

(4) If $^nC_n = 1$ then 0! = 1 ( )

(5) If $P = 2 + \sqrt{3}$, then $\frac{1}{p} = 2 - \sqrt{3}$. ( )

(6) The degree of the equation $3x^5 + xyz^2 + y^3$ is 3 ( )

Answer: I (a)

(1) 4 (Option c)

(2) 4 (Option a)

(3) $\because x^2 - 3x + m = 0$

Let the roots be $\propto$, $\propto + 5$

$\because \propto + (\propto + 5) = 3$

$2 \propto = -2$

$\propto = -1$

$\therefore$ the roots be -1, 4

$\therefore$ Product of roots = M = -4 (Option b)

(4) $\because {}^nP_3 = 120$ or $\frac{\lfloor n}{\lfloor n-3} = 120$

$\Rightarrow n(n-1)(n-2) = 120 = 6 \times 5 \times 4$

$\therefore n = 6$ (Option c)

(5) 25 (Option c)

(6) $\because {}^rc_{12} = {}^rc_8 \Rightarrow r = 12 + 8 = 20.$

$\because {}^{22}c_y = {}^{22}c_{20} = \frac{\lfloor 22}{\lfloor 20 \lfloor 2} = \frac{22 \times 21}{2} = 21 \times 11 = 231$ (Option c)

(7) A : B = 2 : 3

B = (72/2) × 3 = 108 (Option c)

(8) 0 (Option c)

(9) 8 (Option b)

Answer: I (b)

(1) The average of 50 numbers is 38. If two numbers, namely 45 and 55 are discarded, the average of the remaining numbers is 30. (F)

(2) The value of ${}^5C_2$ is equal to ${}^5C_3$. (T)

(3) The geometric mean of 3 and 1/3 is – 1 (F)

(4) If ${}^nC_n = 1$ then 0! = 1 (T)

(5) If $P = 2 + \sqrt{3}$, then $\frac{1}{p} = 2 - \sqrt{3}$. (T)

(6) The degree of the equation $3x^5 + xyz^2 + y^3$ is 10 (F)

II. Answer any four questions. Each question carries 4 marks (4 × 4 = 16)

(1) If x ∝ y, then prove that $x^2 + y^2 \propto x^2 - y^2$.

(2) What sum of money will yield ₹ 1,407 as interest in 1½ year at 14% p.a. simple interest?

(3) Find the value of $\log_5 3 \times \log_3 625$

(4) Find the number of ways in which a person can invite his 4 friends selecting at least 1

(5) From a company of 15 men, how many selections of 9 men can be made so as to exclude 3 particular men?

(6) If p and q are the roots of the quadratic equation $x^2 + x - 1 = 0$, find the value of $\frac{1}{p} + \frac{1}{q}$

Answer: II

(1) $x \alpha y \Rightarrow x = ky$

Now, $\frac{x^2 + y^2}{x^2 - y^2} = \frac{k^2 + 1}{k^2 - 1} \Rightarrow x^2 + y^2 \alpha x^2 - y^2$

(2) What sum of money will yield ₹ 1,407 as interest in 1 year at 14% p.a. simple interest.

Here S.I = 1407, n = 1.5, I = 0.14, P = ?

S.I = P. i.n or, 1407 = p x 0.14 x1.5

Or, $p = \frac{1407}{0.14 \times 1.5} = \frac{1407}{0.21} = 6700$

Required amount = ₹ 6,700

(3) $\log_5 3 \times \log_3 625$

$= \log_5 3 \times \log_3 5^4$

$= 4 \log_5 3 \times \log_3 5$

$= 4 \times 1 = 4.$

(4) ${}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4$

$= 4 + 6 + 4 + 1 = 15.$

(5) Excluding 3 particular men in each case, we are to select 9 men out of (15-3) men. Hence the number of selection is equal to the number of combination of 12 men taken 9 at a time which is equal to

${}^{12}C_9 = \frac{12!}{9!3!} = 220$

(6) $\frac{1}{p} + \frac{1}{q} = \frac{p+q}{pq} = \frac{-b/a}{c/a}$

$= \frac{-1}{\frac{1}{1}} = 1$

Section – B

III. (a) Choose the correct answer **(12 × 2 = 24)**

(1) Given a table:

Value (greater than)	100	150	200	250
Frequency	50	32	9	0

The number of observations between 150 and 200 is

(a) 23 **(b) 15** **(c) 8** **(d) 18**

(2) Given the table:

Class Interval	10-15	15-20	20-25	25-30	30-35
Frequency	5	7	4	6	8

The class mark for the second class is

(a) 27.5 **(b) 17.5** **(c) 12.5** **(d) 32.5**

(3) The Arithmetic Mean for the series 2, 6, 7, 9, 4, 2 is

(a) 7 **(b) 4** **(c) 5** **(d) 6**

(4) $x = \frac{31}{6} - \frac{y}{6}$ is the regression equation of

(b) y on x (b) x on y (c) both (d) none

(5) The mean of first 10 even number is

(a) 5.5 (b) 55 (c) 11 (d) none of these

(6) A. M. of 40 observations of a variable is 25 and S. D. is 4. The sum of squares of all observations is

(a) 25640 (b) 26540 (c) 24560 (d) 26450

(7) If y = a + bx, then what is the co-efficient of correlation between x and y?

(a) 1 (b) -1 (c) 1 or -1 according as b > 0 or b < 0 (d) None of these

(8) The Standard Deviation for two observations is

(a) Square of their difference. (b) half of their absolute difference.

(c) their absolute difference. (d) twice of their absolute difference.

(9) If an unbiased coin is tossed twice, the probability of obtaily of obtaining at least one tail is

(a) 0.25 (b) 0.50 (c) 0.75 (d) 1.00

(10) Difference between the maximum & minimum value of a given data is called –

(a) Width (b) Size (c) Range (d) Class

(11) Two dice are thrown together. The probability that 'the event the difference of nos. shown is 2' is

(a) 2/9 (b) 5/9 (c) 4/9 (d) 7/9

(12) If A and B be two mutually exclusive events and P (A) = $\frac{3}{4}$, P(A∪B) = $\frac{5}{6}$ then P(B) is

(a) $\frac{2}{3}$ (b) $\frac{3}{5}$ (c) $\frac{5}{12}$ (d) $\frac{1}{12}$

III. (b) State whether the following statements are true or false (12 × 1 = 12)

(1) In any group frequency distribution class intervals are of equal width always. ( )

(2) Frequency densities are necessary for drawing histogram. ( )

(3) The slope of the regression line of y on x is b_{yx} ( )

(4) If events are mutually exclusive then their probabilities are less than one ( )

(5) In a moderately asymmetrical distribution A.M. < G.M. < H.M. ( )

(6) Median is the value of a variable which divides the whole statistical data into two parts. ( )

(7) The sum of individual observations from mean is one ( )

(8) If x and y satisfy the relationship y = -5 +7x, the value of r is zero ( )

(9) When one regression coefficient is negative and other regression coefficient is also negative. ( )

(10) Mode is the value that has maximum frequency ( )

(11) In the line $y = 19 - \frac{5x}{2}$, b_{yx} is equal to -5/2 ( )

(12) Sum of all probabilities is equal to one ( )

Answer: III (a)

(1) (a)

(2) (b)

(3) (c)

(4) (b)

(5) (c)

(6) (a)

(7) (c)

(8) (b)

(9) (c)

(10) (c)

(11) (a)

(12) (d)

Answer: III (b)

(1) (F)

(2) (T)

(3) (T)

(4) (F)

(5) (T)

(6) (T)

(7) (F)

(8) (F)

(9) (T)

(10) (T)

(11) (T)

(12) (T)

IV. Answer any four questions. Each question carries 6 marks (4 × 6 = 24)

(1) The frequency distribution of marks of 100 students is given below. If the median is 32, obtain the missing frequencies.

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
No. of students	10	-	25	30	-	10

(2) For 10 values of x, it is given that $\sum u = 4$ and $\sum u^2 = 144$, where $u = \frac{x-10}{5}$. Find the mean and S.D. of x.

(3) The lines of regression of y on x and x on y are respectively y = x + 5 and 16x = 9y — 94. Find the variance of x if the variance of y is 16. Also, find the covariance of x and y.

(4) Compute rank correlation from the following table

X	415	434	420	430	424	428
Y	330	332	328	331	327	325

(5) The means of samples of sizes 50 and 75 are 60 and x respectively. If the mean of the combined group is 54, find x.

(6) Two unbiased dice are thrown. Construct the set of pairs of points having difference 2 between them. Hence, find the probability of getting the difference 2 between the points in each pair.

Answer: IV

(1) Given median = 32

So, median class is the class 30 – 40

Let the missing frequencies be x and y respectively.

$$\text{Now, Median} = l_1 + \frac{\frac{N}{2} - C}{f_m} \times i$$

$$\Rightarrow 32 = 30 + \frac{50 - 35 - x}{30} \times 10$$

$6 = 15 - x \quad \therefore x = 9$

Again $x + y = 100 - 75 \quad \therefore y = 25 - 9 = 16$

(2) Given $\sum u = 4$ when $u = \frac{x-10}{5}$

$\Rightarrow \sum(x - 10) = 5 \times 4 \Rightarrow \sum x - 10 \times 10 = 20$

$\Rightarrow \sum x = 120 \quad \therefore$ Mean of $(x) = \frac{120}{5} = 12$

and $\sum u^2 = 144$

$\Rightarrow \sum (x - 10)^2 = 144 \times 25$

$\Rightarrow \sum x^2 - 20 \sum x + 100 \times 10 = 3600$

$\Rightarrow \sum x^2 = 2600\ 20 \times 120 = 5000$

$$\therefore \text{S. D. } (x) = \sqrt{\frac{\sum x^2}{10} - \left(\frac{\sum x}{10}\right)^2}$$

$= \sqrt{500 - 144} = \sqrt{356} = 18.9$

Other wise

$$\text{Var } (u) = \frac{144}{10} - \left(\frac{4}{10}\right)^2 = 14.24$$

S. D. (u) = 3.7736

S. D. (u) = S. D. (x)/5 (standard deviation is independent of change of origin)

Or, S. D. (x) = 18.9

(3) The lines of regression of y on x and x on y are respectively y = x + 5 and 16x = 9y — 94. So, the regression coefficients of y on x on y are respectively

$b_{yx} = 1$ and $b_{xy} = \frac{9}{16}$

Now, $r^2\, b_{yx} \cdot b_{xy} = 1.\frac{9}{16} = \left(\frac{3}{4}\right)^2$

$\therefore$ Correlation coefficient $(r) = \frac{3}{4}$ (negative value is discarded as regression coefficients are positive)

Again $b_{xy} = r\frac{\sigma_x}{\sigma_y}$, Given variance of y = 16 $\therefore \quad \sigma_y = 4$

$\Rightarrow \frac{9}{16} = \frac{3}{4} \cdot \frac{\sigma_x}{4} \Rightarrow \sigma_x = 3$

So variance of x = 9

Covariance of x and y is Cov. $(x, y) = r\sigma_x \sigma_y = \frac{3}{4}.3.4 = 9$

(4)

X	R_1	Y	R_2	$(R_1 - R_2) = D$	D^2
415	6	330	3	3	9
434	1	332	1	0	0
420	5	328	4	1	1
430	2	331	2	0	0
424	4	327	5	-1	1
428	3	325	6	-3	9

$$r_k = 1 - \frac{6\sum D^2}{N(N^2 - 1)}$$

$$= 1 - \frac{1(20)}{6(6^2 - 1)} = 1 - \frac{120}{210} = \frac{210 - 120}{210} = \frac{90}{210} = \frac{3}{7} = 0.429$$

(5) We have $\overline{x}_{12} = \frac{n_1\overline{x}_1 + n_1\overline{x}_1}{n_1 + n_2}$ or, $54 = \frac{50 \times 60 \times 75x}{50 + 75}$ or, $54 = \frac{3000 \times 75x}{125}$

or, $3000 + 75x = 6750$ or, $75x = 3750$ or, $x = 50$.

(6) Set of favourable pairs: {(1,3),(3,1),(2,4),(4,2), (3,5), (5,3), (4,6),(6,4)}

Total No. of pairs like (1,1),(1,2), ……,(5,6),(6,6) is 36.

Required probability

$$= \frac{\text{No. of Favourable pairs}}{\text{Total no. pairs}} = \frac{8}{36} = \frac{2}{9}$$