Paper 4-Fundamentals of Business Mathematics and Statistics

SET - 2

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Full Marks: 100 **Time allowed: 3 Hours**

The figures in the margin on the right side indicate full marks.

This question paper has two sections.
Both the sections are to be answered subject to instructions given against each.

Section – A
(Business Mathematics)

I. (a) Choose the correct answer **[9 × 2 = 18]**

1. **Two numbers are in the ratio 3:4. If 10 is subtracted from both of them the ratio will be 1:2. So the numbers are**
   (a) 15 and 20 **(b) 12 and 16** **(c) 30 and 40** **(d) None of them**

2. **A person deposits ₹ 2,000 @ 6% p.a. simple interest for 3 years. The amount he will get back after 3 years is**
   (a) 2300 **(b) 2400** **(c) 2360** **(d) 2350**

3. **The value of $\log_2 (\log_5 625)$ is**
   (a) 2 **(b) 5** **(c) 10** **(d) 15**

4. **If $^{n}p_3 = 120$ then n = _____**
   (a) 8 **(b) 4** **(c) 6** **(d) None of these**

5. **If one roots of the equation $x^2 – 3x + m = 0$ exceeds the other by 5 then the value of M is equal to _____**
   (a) -6 **(b) -4** **(c) 12** **(d) 18**

6. **If $^{r}c_{12} = {}^{r}c_8$ find $^{22}c_r$**
   (a) 213 **(b) 321** **(c) 231** **(d) None of these**

7. **The number of ways in which letters of the word Monday be arranged beginning with the letter O and ending the letter Y is**
   (a) 120 **(b) 24** **(c) 96** **(d) None of these**

8. **If the roots of the equation $3/4x^2 + 9x + c^3 = 0$, are equal then C is equal to ……**
   (a) (7) **(b) (6)** **(c) 9** **(d) (3)**

9. **If x varies inversely with Y and if Y = 3, then X = 8. The value of Y when X = 2 are:**
   (a) 24 **(b) 18** **(c) (2, 4)** **(d) None of these**

(b) State whether the following statements are true or false (6×1=6)

(1) The average of 50 numbers is 38. If two numbers, namely 45 and 55 are discarded, the average of the remaining numbers is 36.5 ( )

(2) If 15% of x = 20% of y then x : y = 4 : 3 ( )

(3) The logarithm of one to any base is zero ( )

(4) The Statement "Equivalent sets are always equal" is True or False ( )

(5) The number of different number of 6^{th} digits (without repetition) can be formed form the digits 3, 1, 7, 0, 9, 5 is 120 ( )

(6) The degree of the equation $3x^5 + xyz^2 + y^3$ is 3 ( )

Answer:

(a) (1) (a)

(2) (c)

(3) (a)

(4) (c)

(5) (b)

(6) (c)

(7) (b)

(8) (b)

(9) (c)

(b) (1) (F)

(2) (T)

(3) (T)

(4) (F)

(5) (F)

(6) (F)

II. Answer any four questions. Each question carries 4 marks [4 × 4 = 16]

1. **The ratio of present age of mother or her daughter is 5 : 3. Ten years hence the ratio would be 3 : 2. Find their precentages.**

2. **What sum of money will yield ₹ 1,407 as interest in 1½ year at 14% p.a. simple interest**

3. **Insert 4 arithmetic means between 4 and 324.**

4. **If** $\frac{\log x}{y^2+z^2+yz} = \frac{\log y}{z^2+x^2+zx} = \frac{\log z}{x^2+y^2+xy}$
   Show that $x^{y-z}\, y^{z-x}\, z^{x-y} = 1$

5. **From a company of 15 men, how many selections of 9 men can be made so as to exclude 3 particular men?**

6. **If a, 4, b are in AP and a, 2, b are in G.P., then prove that 1/a + 1/b = 2.**

Answer: II

(1) Let present age of mother be 5x and that of her daughter be 3x years.
10 years hence age of mother will be (5x + 10) years and that of daughter be (3x + 10) years. .
By question $\frac{5x+10}{3x+10} = \frac{3}{2}$ or , 2(5x + 10) = 3(3x + 10) or, 10x + 20 = 9x + 30 or, x = 10
∴ Reqd. ages are 5 × 10 = 50 years and 3 × 10 = 30 years.

(2) What sum of money will yield ₹ 1,407 as interest in 1 year at 14% p.a. simple interest.
Here S.I = 1407, n = 1.5, I = 0.14, P = ?
S.I = P. i.n or, 1407 = p x 0.14 x1.5
Or, $p = \frac{1407}{0.14\times1.5} = \frac{1407}{0.21} = 6700$
Required amount = ₹ 6,700

(3) Let a = 4, b = 324
$d = \left(\frac{b}{a}\right)^{\frac{1}{x+1}} = \left(\frac{239}{4}\right)^{\frac{1}{5}} = (81)^{\frac{1}{3}}$
$\because tn = b$
$\Rightarrow a + (n+1)\, d = b$
$d = \frac{b-a}{n+1} = \frac{324-4}{5} = \frac{320}{5}$ 64
t_1, = 68, t_2 = 132, t_3 = 196, t_4 = 260

(4) $\frac{\log x}{y^2+z^2+yz} = \frac{\log y}{z^2+x^2+zx} = \frac{\log z}{x^2+y^2+xy} = k$ (say)

Or $\log x = k(y^2 + z^2 + yz)$, $\log y = k(z^2 + x^2 + zx)$, $\log z = k(x^2 + y^2 + xy)$ ……(i)

To show $x^{y-z}\, y^{z-x}\, z^{x-y} = 1$, taking logarithm both sides

Log $(x^{y-z} . y^{z-x}. z^{x-y}) = \log 1 = 0$ i. e. to show

$(y - z) \log x + (z - x) \log y + (x- y) \log z = 0$

L. H. S. $= (y - z) . k. (y^2 + z^2 + yz) + (z - x). k. (z^2 + x^2 + zx) + (x - y). k. (x^2 + y^2 + xy)$

$= k(y^3 - z^3 - x^3 + x^3 - y^3) = k. 0 = 0$, hence proved.

(5) Excluding 3 particular men in each case, we are to select 9 men out of (15-3) men. Hence the number of selection is equal to the number of combination of 12 men taken 9 at a time which is equal to

$^{12}C_9 = \frac{12!}{9!3!} = 220$

(6) If a,4 b are in A.P. and a, 2, b are in G.P., then Prove that $\frac{1}{a} + \frac{1}{b} = 2$

$\therefore$ a, 4, b are in A.P., then 4-a = b-4

Or, a+b = 8 …..(1)

Again a, 2, b are in G.P., then $\frac{2}{a} = \frac{b}{2}$

Or, ab = 4 ……(2)

Dividing (1) by (2), we get,

$\frac{a}{ab} + \frac{b}{ab} = \frac{8}{4}$

Or, $\frac{1}{b} + \frac{1}{a} = 2$

Or, $\frac{1}{a} + \frac{1}{b} = 2$

Section - B

III. (a) Choose the correct answer **[12 ×2 = 24]**

1. **The mean of first 10 even number is**
   (a) 5.5 (b) 55 (c) 11 (d) None of these

2. **If the coefficient of correlation between X and Y is 0.48 and covariance is 39, the variance of Y is 25, the standard deviation of X will be …….**
   (a) 14.46 (b) 16.25 (c) 12.80 (d) 9.86

3. **Mode is the value which**
   (a) is a mid point (b) Occur the most likely (c) average of all (d) most remote

4. **$x = \frac{31}{6} - \frac{y}{6}$ is the regression equation of**
   (a) y on x (b) x on y (c) both (d) None

5. **For the observations 6, 4, 1, 6, 5, 10, 4, 8 range is**
   (a) 10 (b) 9 (c) 8 (d) None

6. **For two positive observations x^1 and x^2 which one of the following is true?**
   (a) $(AM)(HM) = (GM)^2$ (b) $(AM)(GM) = (HM)^2$
   (c) $(GM)(HM) = (AM)^2$ (d) None of above

7. **Quartiles are values dividing a given set of data into …….. equal parts**
   (a) 4 (b) 6 (c) 3 (d) 2

8. **The harmonic mean for the series 6, 5, 3, 6, 7, 10 and 12 is**
   (a) 5.87 (b) 6.21 (c) 5.12 (d) 5.98

9. **A card is drawn from a pack of 52 cards. The probability of getting a Queen is ……**
   (a) 1/4 (b) 1/13 (c) 3/13 (d) 2/13

10. **If y = a + bx, then what is the co-efficient of correlation between x and y?**
    (a) 1 (b) -1 (c) 1 or -1 according as b > 0 or b < 0 (d) None of these

11. **Two dice are thrown together. The probability that 'the event the difference of nos. shown is 2' is**
    (a) 2/9 (b) 5/9 (c) 4/9 (d) 7/9

12. **A pair of dice is thrown. The probability of getting a total of 8 is ……..**

(a) 2/26 (b) 1/4 (c) 1/3 (d) 5/36

(b) State whether the following statements are true or false (12×1=12)

(1) There is no difference between co-efficient of variation and variance ( )

(2) Sum of probability of an event A and its complements is ( )

(3) The slope of the regression line of y on x is b_{xy} ( )

(4) If events are mutually exclusive then their probabilities are less than one ( )

(5) In a moderately asymmetrical distribution AM. < G.M. < H.M. ( )

(6) Median can never be equal to mean in a skewed distribution ( )

(7) The sum of individual observations from mean is zero ( )

(8) If x and y satisfy the relationship y = -5 + 7x, the value of r is zero ( )

(9) In a normal distribution SD > MD > QD ( )

(10) Mode is the value that has maximum frequency ( )

(11) In the line $y = 19 - \frac{5x}{2}$, b_{xy} is equal to -5/2 ( )

(12) Sum of all probabilities is equal to one ( )

Answer:

(a) (1) (c)

(2) (b)

(3) (b)

(4) (b)

(5) (b)

(6) (a)

(7) (a)

(8) (a)

(9) (b)

(10) (c)

(11) (a)

(12) (d)

(b) (1) (F)

(2) (T)

(3) (T)

(4) (F)

(5) (T)

(6) (T)

(7) (T)

(8) (F)

(9) (T)

(10) (T)

(11) (T)

(12) (T)

IV. Answer any four questions. Each question carries 6 marks [4 × 6 = 24]

(1) Draw the histogram of the following data and comment on the shape of the distribution.

Wages (in ₹)	50-59	60-69	70-79	80-89	90-99
No. of employees	8	10	16	12	7

(2) The mean and standard deviation of the marks obtained by the groups of the students consisting of 50 each are given below:

Group	Mean	S.D.
A	60	8
B	55	7

Calculate the mean and standard deviation of the marks obtained by all 100 students.

(3) The marks obtained by 6 students were 24, 12, 16, 11, 40, 42. Find the range. If the highest mark is omitted, find the percentage change in the range.

(4) Find the median of the following frequency distribution:

Value (x):	1	2	3	4

Frequency (f):	7	12	18	4

(5) The means of samples of sizes 50 and 75 are 60 and x respectively. If the mean of the combined group is 54, find x.

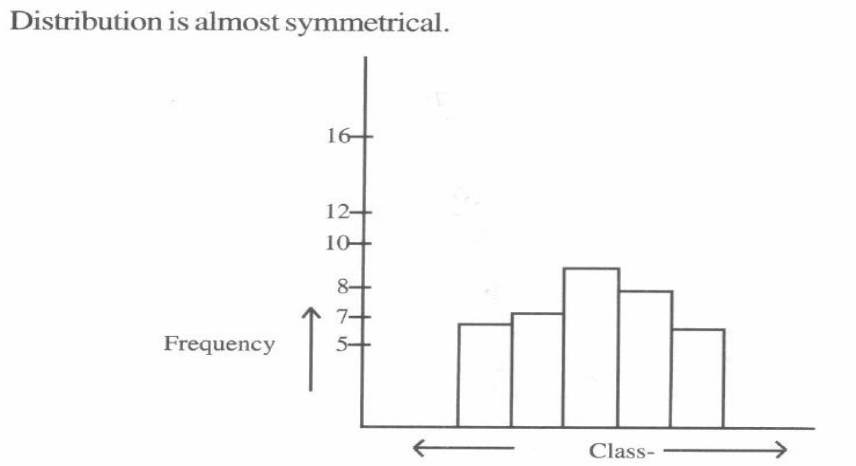
(6) What is the chance of throwing more than 15 in one throw with three dice?

Answer: IV

(1)

Class- boundaries	:	49.5- 59.5	59.5 – 69.5	69.5 – 79.5	79.5 – 89.5	89.5-99.5
Frequency	:	8	10	16	12	7

HISTOGARM:



(2) Here $n_1 = 50$, $n_2 = 50$, $\bar{X}_1 = 60$, $\bar{X}_2 = 55$

So, $\bar{X} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1+n_2} = \dfrac{50\times60+50\times55}{100} = \dfrac{300+2750}{100} = \dfrac{5750}{100} = 57.50$

Now $d_1 = \bar{X}_1 - \bar{X} = 60 - 57.5 = 2.5$, $d_2 = \bar{X}_2 - \bar{X} = 55 - 57.5 = -2.5$

$S_1^2 = 64$, $S_2^2 = 49$

Hence, $S^2 = \dfrac{n_1(S_1^2+d_1^2)+n_2(S_2^2+d_2^2)}{n_1+n_2}$

$= \dfrac{50[64+6.25+49+6.25]}{100} = \dfrac{50\times125.50}{100} = 62.75$

$S = \sqrt{62.75} = 7.92$

Hence, Mean $\bar{X} = 57.5$ and std deviation (s) = 7.92.

(3) The marks obtained by 6 students were 24, 12, 16, 11, 40, 42. Find the Range. If the highest mark is omitted, find the percentage change in the range.
Here maximum mark = 42, minimum mark = 11.

$\therefore$ Range = 42 - 11 = 31 marks

If again the highest mark 42 is omitted, then amongst the remaining. Maximum mark is 40. So, i (revised) = 40 - 11 =29 marks.

Change in range = 31 - 29 = 2 marks.
∴ Reqd. percentage change = 2 ÷ 31 × 100 = 6.45%

Note: Range and other obsolute measures of dispersion are to be expressed in the same unit in which observations are expressed.

For grouped frequency distribution:
In this case range is calculated by subtracting the lower limit of the lowest class interval from the upper limit of the highest.

(4)

x	f	c.f. (less than type)
1	7	7
2	12	19
3	18	37
4	4	41 (N)

Median = Value of $\frac{N+1}{2}$ th item
= Value of $\frac{41+1}{2}$ th item
= value of 21st item
= 3

(5) We have $\overline{x_{12}} = \frac{n_1\overline{x_1} + n_1\overline{x_1}}{n_1 + n_2}$ or, $54 = \frac{50 \times 60 \times 75x}{50+75}$ or, $54 = \frac{3000 \times 75x}{125}$

or, 3000 + 75x = 6750 or, 75x = 3750 or, x = 50.

(6) Total number of cases = 6 x 6 X 6 = 216
Throwing more than 15 means getting 16, 17 or 18.
Possible ways of throwing 16 are (6, 6, 4), (6, 5, 5) (6, 4, 6), (5, 5, 6) (5, 6, 5) **and** (4, 6, 6).
Number of favourable cases = 6.

The probability of getting 16 with three dice = $\frac{6}{216}$

Possible ways of throwing 17 are (6, 6, 5), (6, 5, 6) and 5, 6, 6

3 The probability of throwing 17 with three dice = $\frac{6}{216}$

There is only one way of throwing 18 with three dice namely (6,6,6).

The probability of throwing 18 with three dice = $\frac{6}{216}$

The three cases are mutually exclusive.
Therefore the probability of throwing more than 15.

$= \frac{6}{216} + \frac{3}{216} + \frac{1}{216} = \frac{10}{216} = \frac{5}{108}$