Paper 4-Fundamentals of Business Mathematics and Statistics

<u>SET - I</u>

Paper 4-Fundamentals of Business Mathematics and Statistics

Full Marks: 100   **Time allowed: 3 Hours**

The figures in the margin on the right side indicate full marks.
This question paper has two sections.
Both the sections are to be answered subject to instructions given against each.

Section – A
(Business Mathematics)

I.   (a) Choose the correct answer   [9 × 2 = 18]

   1.   The number to be added to each term of the ratio 3 : 7 to make it 1 : 2 is
        (a) 2   (b) 1   (c) 3   (d) None of these

   2.   At what rate p.a. S.I. will a sum of money double itself in 25 years?
        (a) 4%   (b) 3%   (c) 5%   (d) 6%

   3.   If A : B = 3 : 4 & B : C = 2 : 5, then A : B : C
        (a) 3 : 4 : 5   (b) 3 : 4 : 10   (c) 4 : 3 : 10   (d) 3 : 4 : 8

   4.   A.M. of two integral numbers exceeds their G.M. by 2 and the ratio of the numbers is 1 : 4.
        Find the numbers.
        (a) 5, 20   (b) 1, 4   (c) 2, 8   (d) 4, 16

   5.   Set of even positive integers less than equal to 6 by selector method.
        (a) {x/x<6}   (b) {x/x=6}   (c) {x/x≤6}   (d) None

   6.   The value of $\log^{32}_{\sqrt{2}}$ is

        (a) 5/2   (b) 5   (c) 10   (d) 1/10

   7.   If $^{n}p_{3}$ = 120 then n = _____
        (a) 8   (b) 4   (c) 6   (d) None of these

   8.   If $^{r}c_{12}$ = $^{r}c_{8}$ find $^{22}c_{r}$
        (a) 213   (b) 321   (c) 231   (d) None of these

   9.   If $3x^2 +6x +3 = 0$, then roots of the equation are
        (a) (3, 3)   (b) (-1, -1)   (c) (2, 4)   (d) (4, 1)

   (b) State whether the following statements are true or false   (6×1=6)

        (1) If 15% of x = 20% of y then x : y = 4 : 3   (   )

(2) If the terms -1 + 2x, 5, 5+x are is an A.P. then x is 4 **( )**

(3) The statement "Equivalent sets are always equal" is true or false **( )**

(4) The logarithm of one to any base is zero **( )**

(5) $^nc_0$ = 1 is true or false **( )**

(6) The degree of the equation $3x^5 + xyz^2 + y^3$ is 3 **( )**

Answer:

(a) (1)   (b)

(2)   (a)

(3)   (b)

(4)   (d)

(5)   (c)

(6)   (c)

(7)   (c)

(8)   (c)

(9)   (b)

(b) (1)   (T)

(2)   (F)

(3)   (F)

(4)   (T)

(5)   (T)

(6)   (F)

II. Answer any four questions. Each question carries 4 marks [4 × 4 = 16]

1. If $\dfrac{x}{b+c} = \dfrac{y}{c+a} + \dfrac{z}{a+b}$ then show that $(b-c)(x-a) = (c-a)(y-b) = (a-b)(z-c) = 0$.

2. Which is better investment – 3% per year compounded monthly (or) 3.2% per simple interest (given that $(1.0025)^{12} = 1.0304$)

3. Insert 4 arithmetic means between 4 and 324.

4. Prove that $\dfrac{\log\sqrt{27} + \log 8 + \log\sqrt{100}}{\log 14400} = \dfrac{3}{4}$

5. Prove that CALCUTTA is twice of AMERICA in respect of number of arrangements of letters.

6. If the roots of the equation $ax^2 + bx + c = 0$ in the ratio 2 : 3, then show that $6b^2 = 25ca$.

Answer:

1. Let $\dfrac{x}{b+c} = \dfrac{y}{c+a} + \dfrac{z}{a+b}$ = k (constant). Say

   Then x = K(b+c), y = k(c+a), z = a(a+b)

   So, (b-c)(x-a)+(c-a)(y-b)+(a-b)(z-c)
   = [x(b-c)+y(c-a)+z(a-b)]-[a(b-c)+b(c-a)+c(a-b)]
   = [k(b+c)(b-c)+k(c+a)(c-a)(c-a)+k(a+b)(a-b)]-[ab+ac+bc-ab+ac-bc]
   = [k(b²-c²)+k(c²-a²)+k(a²-b²)]-0
   = [k(b²-c²+c²-a²+a²-b²)]-0
   = k×0-0=0-0=0 Proved

2. ∴ ₹200, ₹280

   $r_e = 100\left\{(\dfrac{1+i}{m})^m - 1\right\}$

   $= 100\left[\left(\dfrac{1+3}{1200}\right)^{12} - 1\right]$

   $= 100\left[\left(\dfrac{1203}{1200}\right)^{12} - 1\right]$

   $= 100\,(0.304)$

   $= 3.04\%$

   ∴ 3.2% S. I in better investment.

3. Let $a = 4$, $\qquad\qquad\qquad$ $b = 324$

$$d = \left(\frac{b}{a}\right)^{\frac{1}{x+1}} = \left(\frac{239}{4}\right)^{\frac{1}{5}} = (81)^{\frac{1}{3}}$$

$\because t_n = b$

$\Rightarrow a + (n+1) d = b$

$$d = \frac{b-a}{n+1} = \frac{324 - 4}{5} = \frac{320}{5} \quad 64$$

$t_1, = 68, t_2 = 132, t_3 = 196, t_4 = 260$

4. $\dfrac{\log\sqrt{27} + \log 8 + \log\sqrt{100}}{\log 14400}$

$= \dfrac{\log 3^{3/2} + \log 2^3 + \log 10^{3/2}}{\log(120)^2}$

$= \dfrac{\frac{3}{2}\log 3 + 3\log 2 + \frac{3}{2}\log 10}{2\log 120}$

$= \dfrac{\frac{3}{2}(\log 3 + 2\log 2 + \log 10)}{2\log(3 \times 4 \times 10)}$

$= \dfrac{3(\log 3 + \log 4 + \log 10)}{4(\log 3 + \log 4 + \log 10)}$

$= \dfrac{3}{4} = $ R.H.S.

5. In CALCUTTA, each of the letters C, A, T occurs twice and total number of letter is 8.

So, number of arrangements is $\dfrac{8!}{2!2!2!} = \dfrac{8 \times 7!}{8} = 7!$

And in AMERICA, A occurs twice and total number of letters is 7, so no. of arrangements is $\dfrac{7!}{2}$.

Hence in respect of arrangements of letters, CALCUTTA is twice of AMERICA.

6. Let α, β be the roots of the equation $ax^2 + bx + c = 0$ so that

$\alpha + \beta = \dfrac{-b}{a}$ .... (i); $\qquad\qquad$ $\alpha\beta = \dfrac{c}{a}$ .... (ii)

Again, $\dfrac{\alpha}{\beta} = \dfrac{2}{3}$; or $\qquad\qquad$ $\alpha = \dfrac{2}{3}\beta$ .... (iii)

From (i), $\dfrac{2}{3}\beta + \beta = \dfrac{-b}{a}$; or $\qquad \dfrac{5\beta}{3} = \dfrac{-b}{a}$ or, $\qquad \beta = \dfrac{3}{5} \times \dfrac{-b}{a} = \dfrac{-3b}{5a}$

From (iii), $\alpha = \dfrac{2}{3} \times \dfrac{-3b}{5a} = \dfrac{-2b}{5a}$

From (ii), $\dfrac{-2b}{5a} \times \dfrac{-3b}{5a} = \dfrac{c}{a}$; $\qquad\qquad$ or, $\dfrac{6b^2}{25a^2} = \dfrac{c}{a}$; $\qquad\qquad$ or, $6b^2 = 25ac$ [as, $a \neq 0$].

Section – B

III. (a) Choose the correct answer [12 ×2 = 24]

1. If the co-efficient of correlation between two variables is 0.8 and their covariance is 24. If the variance of x series on Y is 25, the standard deviation of Y series will be ……………
   (a) 6.24   (b) 5.94   (c) 6.00   (d) 5.54

2. Which of the following measures of averages divide the observation into four equal parts
   (a) Mean   (b) Median   (c) Mode   (d) Quartile

3. A variable which can assume any value between two given value is called
   (a) Continuous   (b) Discrete Value   (c) Random   (d) None

4. Class mark is
   (a) A midpoint of class interval   (b) Upper point of class interval
   (c) Lower class   (d) None

5. For the observations 6, 4, 6, 5, 10, 4, 8 range is
   (a) 10   (b) 9   (c) 8   (d) None

6. Harmonic mean is used for calculating
   (a) Average Growth Rate of Variables
   (b) Average speed of journey
   (c) Average rate of increase in net worth of a company
   (d) All the above 1 to 3

7. $x = \frac{31}{6} - \frac{y}{6}$ is the regression equation of

   (a) y on x   (b) x on y   (c) both   (d) None

8. Two regression lines coincide when
   (a) r = 0   (b) r = 2   (c) r = +1 or -1   (d) None

9. The mode for the series 3, 5, 6, 2, 6, 2, 9, 5, 8, 6 is
   (a) 5.1   (b) 5   (c) 6   (d) 8

10. Standard deviation is used to measure the
    (a) Degree of variation or uniformity in data   (b) Mode value
    (c) Extent of extremes values   (d) All the three

11. If an unbiased coin is tossed twice, the probability of obtaining at least one tail is
    (a) 0.25   (b) 0.50   (c) 0.75   (d) 1.00

12. Two dice are thrown together. The probability that the event the difference of nos.
    (a) 1   (b) -1   (c) 1 or -1 according as b > 0 or b < 0   (d) None of these

(b) State whether the following statements are true or false **(12×1=12)**

(1) Horizontal bar diagram is used for qualitative data **( )**

(2) Median is a mathematical average **( )**

(3) Co-efficient of variation = $\dfrac{\text{Co - efficient of variation}}{\text{Mean}} \times 100$ **( )**

(4) Range is the value of difference between mode and median **( )**

(5) If a coin is tossed, then probability of getting two heads is zero **( )**

(6) If an unbiased coin is tossed once, then the two events head and tail are mutually exclusive **( )**

(7) 10th percentile is equal to 9th Decile **( )**

(8) Mean deviation can never be negative **( )**

(9) The value of correlation co-efficient lies between 0 & 1 **( )**

(10) Bivariate data are the data collected for two variables **( )**

(11) When all value s are equal, then standard deviation would be zero **( )**

(12) As the sample size increase, range tends to decrease **( )**

Answer:

(a)   (1)   (c)

        (2)   (d)

        (3)   (a)

        (4)   (a)

        (5)   (b)

        (6)   (b)

        (7)   (b)

        (8)   (c)

        (9)   (c)

(10) (a)

(11) (c)

(12) (a)

(b) (1) (T)

(2) (F)

(3) (F)

(4) (F)

(5) (T)

(6) (T)

(7) (F)

(8) (T)

(9) (F)

(10) (T)

(11) (T)

(12) (F)

IV. Answer any four questions. Each question carries 6 marks   **[4 × 6 = 24]**

(1) Prove that for any two positive real quantities AM ≥ GM ≥ HM.

(2) Find the median and mode of the following grouped frequency distribution:

Salaries (in ₹) per hours	5-9	10-14	15-19	20-24	25-29	Total
No. of Persons	10	20	30	25	15	100

(3) The marks obtained by 6 students were 24, 12, 16, 11, 40, 42. Find the range. If the highest mark is omitted, find the percentage change in the range.

(4) Calculate Karl Pearson's coefficient of correlation between variables X and Y using the following data:

X	25	40	30	25	10	5	10	15	30	20
Y	10	25	40	15	20	40	28	22	15	5

(5) The data about the sales and advertisement expenditure of firm are given below:

Commodity	Sales (₹ In crores)	Advertisement Expenditure (₹ In crores)
Mean	40	6
Standard deviation	10	1.5
Coefficient of correlation		0.9

(i) Estimate the likely sales for a proposed advertisement expenditure of ₹ 10 crores.

(ii) What should be the advertisement expenditure if the firm proposes a sales target of ₹ 60 crores?

(6) What is the change that a leap year, selected at random will contain 53 Sundays?

Answer:

(1) Let x_1 and x_2 be any two positive real quantities.

Now $(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2$

=> $(x_1 - x_2)^2 - 4x_1x_2 \geq 0$

=> $\left(\dfrac{X_1 + X_2}{2}\right)^2 \geq x_1x_2$

=> $\dfrac{X_1 + X_2}{2} \geq \sqrt{x_1x_2}$

=> AM ≥ GM ……………(I)

Next $\dfrac{X_1 + X_2}{\frac{X_1X_2}{2}} \geq \dfrac{\sqrt{x_1x_2}}{x_1x_2}$ => $\dfrac{\frac{1}{X_1} + \frac{1}{X_2}}{2} \geq \dfrac{1}{\sqrt{x_1x_2}}$

=> $\dfrac{2}{\frac{1}{X_1} + \frac{1}{X_2}} \geq \sqrt{x_1x_2}$

=> HM ≤ GM ……………………… (II)

Combining (I) & (II)

AM ≥ GM ≥ HM.

(2)

Computation of Mean and Mode

Class (₹)	Frequency	Cumulative Frequency (<type)
4.5 – 9.5	10	10
9.5 – 14.5	20	30
14.5 – 19.5	30	60
19.5 – 24.5	25	85
24.5 – 29.5	15	100

$\dfrac{100}{2}$ = 50. So median class is 14.5 – 19.5 since value corresponding to 50(C.F.) lies in that class.

Median = $14.5 + \dfrac{\frac{100}{2} - 30}{60 - 30} \times 5 = 14.5 + \dfrac{20}{30} \times 5 = 14.5 + \dfrac{10}{3}$ = 14.5+3.33 = 17.83 ₹

Modal class is 14.5 – 19.5 since maximum frequency 30 lies in that class

Mode $= 14.5 + \dfrac{30-20}{(30-20)+(30-25)} \times 5 = 14.5 + \dfrac{10}{10+5} \times 5 = 14.5 + \dfrac{10}{3} = 14.5 + 3.33 = 17.83$ ₹

(3) The marks obtained by 6 students were 24, 12, 16, 11, 40, 42. Find the Range. If the highest mark is omitted, find the percentage change in the range.

Here maximum mark = 42, minimum mark = 11.

∴ Range = 42 - 11 = 31 marks

If again the highest mark 42 is omitted, then amongst the remaining. Maximum mark is 40. So, i (revised) = 40 - 11 = 29 marks.

Change in range = 31 - 29 = 2 marks.

∴ Reqd. percentage change = 2 ÷ 31 × 100 = 6.45%

Note: Range and other obsolute measures of dispersion are to be expressed in the same unit in which observations are expressed.

For grouped frequency distribution:
In this case range is calculated by subtracting the lower limit of the lowest class interval from the upper limit of the highest.

(4) Table: Calculation of Coefficient of correlation

X	Y	X=X-21	Y=Y-22	X^2	Y^2	XY
25	10	4	-12	16	144	-48
40	25	19	3	361	9	57
30	40	9	18	81	324	162
25	15	4	-7	16	49	-28
10	20	-11	-2	121	4	22
5	40	-16	18	256	324	-288
10	28	-11	6	121	36	-66
15	22	-6	0	36	0	0
30	15	9	-7	81	49	-63
20	5	-1	-17	1	289	17
∑X=210	∑Y=220	∑X=0	∑Y=0	∑X^2=1090	∑Y^2=1228	∑XY=-235

$$\bar{X} = \frac{\sum X}{N} = \frac{210}{10} = 21$$

$$\bar{Y} = \frac{Y}{N} = \frac{220}{10} = 22$$

$$r = \frac{-235}{\sqrt{1090 \times 1228}} = \frac{-235}{1156.94} = -0.203$$

(5) Let the sales be denoted by x and advertisement expenditure by Y.

We have $\overline{X} = 40$

$\overline{Y} = 6$

$\sigma_x = 10$

$\sigma_Y = 1.5$

r = 0.9

Regression Equation of Y on X

$Y - \overline{Y} = r \dfrac{\sigma_Y}{\sigma_x} (x - \overline{X})$

(i) To estimate the likely sales for a proposed advertisement expenditure of ₹ 10 crores, we have to find regression equation of X on Y.

Regression Equation of X on Y.

$X - \overline{X} \qquad = r \dfrac{\sigma_x}{\sigma_Y} (Y - \overline{Y})$

$Y - 40 \qquad = 0.9 \dfrac{10}{1.5} (Y - 6)$

$X - 40 \qquad = 6(Y - 6)$

$X - 40 \qquad = 6Y - 36$

$X = 40 + 6Y - 36$

$X = 6Y + 4$

Putting Y = 10 in above equation

$X = 6 \times 10 + 4 = 64$

Hence, estimated sales = ₹ 64 crores.

(ii) To estimate the advertisement expenditure if the firm proposed as sales target of ₹ 60 crores, we find regression equation of Y on X.

Regression Equation of Y on X:

$Y - \overline{Y} = r \dfrac{\sigma_Y}{\sigma_X} (X - \overline{X})$

$Y - 6 = 0.9 \dfrac{1.5}{10} (X - 40)$

$Y - 6 = 0.135 (X - 40)$

$Y - 6 = 0.135 x - 5.4$

$Y = 6 + 0.135X - 5.4$

$Y = 0.6 + 0.135x$

Putting X = 60 in above equation of Y on X.

$Y = 0.6 + 0.135 (60)$

$Y = 0.6 + 8.10$

Y = 8.7 crores.

(6) As a leap year consist of 366 days it contains 52 complete weeks and two more days. The two consecutive days make the following combinations:

(a) Monday and Tuesday

(b) Tuesday and Wednesday

(c) Wednesday and Thursday

(d) Thursday and Friday

(e) Friday and Saturday
(f) Saturday and Sunday, and
(g) Sunday and Monday

If *(f)* or (g) occur, then the year consists of 53 Sundays.

Therefore the number of favourable cases = 2

Total number of cases = 7

The probability = $\dfrac{2}{7}$