Paper- 4: FUNDAMENTALS OF BUSINESS MATHEMATICS AND STATISTICS

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Full Marks: 100 **Time Allowed: 3 Hours**

Section – A

(Fundamentals of Business Mathematics)

PART – A

1. (a) Choose the correct answer from the given four alternatives: [9×2 = 18]

(i) 20 litres of a mixture contain milk and water in the ratio 5 : 3. If 4 litres of this mixture are replaced by 4 litres of milk, the ratio of milk to water in the new mixture will become:

(a) 2 : 1

(b) 6 : 3

(c) 7 : 3

(d) 8 : 3

(ii) $81^{(\log_5 3)^{-1}} + 27^{\log_9 36}\left[3^{(\log_7 9)^{-1}}\right]^4$ **gives:**

(a) 216

(b) 625

(c) 890

(d) 980

(iii) On the average experienced person does 5 units of work, while a fresh one 3 units of work daily but the employer has to maintain an output of at least 30 units of work per day. This situation can be expressed as

(a) $5x + 3y \leq 30$

(b) $5x + 3y \geq 30$

(c) $5x + 3y > 30$

(d) None of these

(iv) The difference between simple interest and compound interest on a sum of money for 2 years at 5% is ₹25. The sum is

(a) ₹ 8,000

(b) ₹ 9,000

(c) ₹ 10,000

(d) ₹ 15,000

(v) In how many ways can 5 beads of different colours form a necklace

(a) 24

(b) $\frac{4!}{2}$

(c) 56

(d) None of these

(vi) Eight guests have to be seated, 4 on each side of a long rectangular table, 2 particular guests desire to sit on one side of the table and 3 others on the another side. The number of ways in which the selection can be made is

(a) 4

(b) 3

(c) 5

(d) None of these

(vii) If the arithmetic mean between two numbers *x* and y is thrice their geometric means, then the ratio *x* : y could be -

(a) $17 + 12\sqrt{2}$

(b) $17 - 12\sqrt{2}$

(c) $17 \pm 12\sqrt{2}$

(d) None of these.

(viii) The ratio between the sum of n terms of two arithmetical progressions is (7n + 1) : (4n + 27). The ratio of their 11th term is -

(a) 124: 105

(b) 136 : 117

(c) 148: 111

(d) None of these.

(ix) If $A \Delta B = (A - B) \cup (B - A)$ and A = {1, 2, 3, 4}, B = {3, 5, 7} then $A \Delta B$ is

(a) {1, 2, 4, 5, 7}

(b) {3}

(c) {1, 2, 3, 4, 5, 7}

(d) None of these

Answer:

(i) (c) 7 : 3

(ii) (c) 890

(iii) (b) $5x + 3y \geq 30$

(iv) (c) ₹ 10,000

(v) (b) $\frac{4!}{2}$

(vi) (b) 3

(vii) (c) $17 \pm 12\sqrt{2}$

(viii) (c) 148 : 111

(ix) (a) {1, 2, 4, 5, 7}

(b) State whether the following statements are True (or) False. [6×1=6]

(i) The ratio of two numbers is 12:5. If antecedent is 45 then the consequent is 108

(ii) The rate of S.I p.a a sum of money grows to one and a half times itself in 8 yrs is 6½%

(iii) The statements "Equivalent sets are always equal is True (or) False

(iv) There are 8 questions in an examination paper and each question has an alternative. The number of ways in which a student can give his answer is 6561.

(v) The value of $\left(\frac{243}{32}\right)^{-4/5}$ is $\frac{81}{16}$.

(vi) The C.I on a certain sum of money for 1 year at 8% p.a compounded quarterly is ₹ 824 then the sum is ₹ 10,000

Answer:

(i) False

(ii) True

(iii) False

(iv) False

(v) False

(vi) True

PART - B

Answer any *four* questions out of six questions: [4×4=16]

2. A Dealer mixes Tea costing ₹ 6.92 per kg. with Tea costing ₹ 7.77 per kg. and sells the mixture at ₹ 8.80 per kg. and earns a profit of 17.5% on his Sale Price. In what proportion does he mix them? [4]

Answer:

Let us first find the Cost Price (CP) of the mixture. If SP is ₹1, profit is 0.175

Therefore, CP = 1 – 0.175 = 0.825.

1. If SP. Is ₹ 8.8, CP is 0.825 × 8.8 = ₹7.26 ∴ C.P. of the mixture per kg = ₹7.26
2. Let X be the proportion of Type A tea and Y be the proportion of Type B tea.

Type of Tea	Cost	Weight	Total Cost
A	6.92/kg	X	6.92X
B	7.77/kg	Y	7.77Y
Total		**X + Y**	**6.92X + 7.77Y**

Hence the Cost per Kg. of the Mixture = $\frac{\text{Total Cost}}{\text{Total Weight}} = 7.26$ **[From 1]** $= \frac{6.92X + 7.77Y}{X + Y} = 7.26$

$= 6.92X + 7.77\ Y = 7.26\ X + 7.26\ Y \Rightarrow 0.51Y = 0.34X \Rightarrow Y = 0.6667X$

3. Since the Cost per kg, of the Mixture is ascertained, we can say that **X + Y = 1.**

From (2), X + 0.6667X = 1, **X = 0.60, Y = 0.40.,** i.e. he mixes X and Y in the proportion of 3:2.

3. A sum of money invested at C.I. payable yearly amounts to ₹ 10,816 at the end of the second year and to ₹ 11,248.64 at the end of the third year. Find the rate of interest and the sum. [4]

Answer:

Here $A_1 = 10{,}816$, n = 2, and $A_2 = 11{,}248.64$, n = 3

From $A = P(1+i)^n$ we get,

$10{,}816 = P(1+i)^2$ ….. (i) and $11{,}248.64 = P(1+i)^3$…… (ii)

Dividing (ii) by (i) $\frac{11{,}248.64}{10{,}816} = \frac{P(1+i)^3}{P(1+i)^2}$ or, $(1+i) = \frac{11{,}248.64}{10{,}816}$

or $i = \frac{11{,}248.64}{10{,}816} - 1 = \frac{432.64}{10{,}816} = .04$, $r = i \times 100 = .04 \times 100 = 4$ ∴ required rate = 4%

Now from (i) $P = \frac{10{,}816}{(1+.04)^2} = \frac{10{,}816}{(1.04)^2}$

Log P = log 10,816 – 2 log (1.04) = 4.034 – 2(0.170) = 4.034 – .0340 = 4.000

P = antilog 4.000 = 10,000 ∴ required sum = ₹ 10,000.

4. How many terms of A.P. 24, 20, 16,... amount to 72. Explain the double answer. [4]

Answer:

Let required number of terms be *n*.

Take $a_1 = 24$, $d = 20 - 24 = -4$, $S_n = 72$.

$S_n = n/2\ [2a_1 + (n-1)(-4)]$

$\Rightarrow 72 = n/2\ [48 + (n-1)(-4)]$

$\Rightarrow 144 = n\ [48 - 4n + 4]$

$\Rightarrow 144 = n\ [52 - 4n]$

$\Rightarrow 144 = 52n - 4n^2$

$\Rightarrow 4n^2 - 52n + 144 = 0$

$\Rightarrow n^2 - 13n + 36 = 0$

$\Rightarrow n^2 - 4n - 9n + 36 = 0$

$\Rightarrow (n-4) - 9(n-4) = 0$

$\Rightarrow (n-4)(n-9) = 0$

$\Rightarrow n = 4, 9$

∴ Sum of the first 4 terms as well as sum of the first 9 terms is 72. It is due to the fact that sum of the 5th to 9lh terms is zero.

5. If $x = \log_{2a} a$, $y = \log_{3a} 2a$, $z = \log_{4a} 3a$, show that : $xyz+1 = 2yz$ [4]

Answer:

L.H.S = $\log_{2a} a . \log_{3a} 2a . \log_{4a} 3a + 1$

= $(\log_{10} a \times \log_{2a} 10).(\log_{10} 2a \times \log_{3a} 10).(\log_{10} 3a \times \log_{4a} 10) + 1$

$= \frac{\log_{10} a}{\log_{10} 2a} \times \frac{\log_{10} 2a}{\log_{10} 3a} \times \frac{\log_{10} 3a}{\log_{10} 4a} + 1$

$= \frac{\log_{10} a}{\log_{10} 4a} + 1 = \log_{4a} a + \log_{4a} 4a = \log_{40} (a \cdot 4a) = \log_{4a} 4a^2$

R.H.S.= $2 \log_{3a} 2a . \log_{4a} 3a^2 = \log_{4a} (2a)^2 = \log_{4a} . 4a^2$

Hence the result.

6. If ${}^nC_r = 56$ and ${}^nP_r = 336$. Find r and n. **[4]**

Answer:

$$ {}^nC_r = \frac{{}^nP_r}{r!} $$

$$ \Rightarrow \quad 56 = \frac{336}{r!} $$

$$ \Rightarrow \quad r! = \frac{\cancel{336}^{\,6}}{\cancel{56}^{\,\cancel{42}\,1}} = 6 $$

$$ = 3 \times 2 \times 1 $$

$$ = 3\,! $$

$$ \therefore \quad r = 3 $$

Now, $ {}^nP_3 = 336 $

$$ \Rightarrow \quad \frac{n!}{(n-3)!} = 336 $$

$$ \Rightarrow \quad n\,(n-1)\,(n-2) = 8 \times 7 \times 6 $$

Comparing both sides, we get n = 8.

7. Solve $\left(x-\frac{1}{x}\right)^2 + 2\left(x+\frac{1}{x}\right) = 7\frac{1}{4}$. **[4]**

Answer:

$$ \left(x-\frac{1}{x}\right)^2 + 2\left(x+\frac{1}{x}\right) = \frac{29}{4} $$

$$ \left(x+\frac{1}{x}\right)^2 - 4 + 2\left(x+\frac{1}{x}\right) = \frac{29}{4} $$

[Since, $(a-b)^2 = (a+b)^2 - 4ab$)]

Substitute $p = x + \frac{1}{x}$;

$p^2 + 2p - 45/4 = 0$; $4p^2 + 8p - 45 = 0$

$$ 4p^2 + 18p - 10p - 45 = 0 $$

$$ 2p(2p+9) - 5(2p+9) = 0 $$

$(2p-5)(2p+9) = 0$

$\therefore 2p+9=0$ or $2p-5=0$

$\Rightarrow p = -\frac{9}{2}$ or $p = \frac{5}{2}$

$\therefore x+\frac{1}{x}=-\frac{9}{2}$ or $x+\frac{1}{x}=\frac{5}{2}$

i.e. $2x^2+9x+2=0$ or $2x^2-5x+2=0$

i.e. $x=\frac{-9\pm\sqrt{81-16}}{4}$ or, $x=\frac{5\pm\sqrt{25-16}}{4}$

i.e. $x=\frac{-9\pm\sqrt{65}}{4}$ or $x = 2$ or $\frac{1}{2}$

Section – B

PART - A

8. Answer All objective questions.

(a) Answer Multiple Choice Question **[12×2= 24]**

(i) The sum of all the relative frequencies in a sample is equal to

(a) the sample size

(b) zero

(c) one

(d) none of these

(ii) The ordinal classification is

(a) The classification of data on the basis of attributes

(b) The classification of data on the basis of numeric characteristics

(c) The classification of data on the basis of qualitative data

(d) both (a) and (c)

(iii) If the mean of two values is 16 and their H.M is 9, then their G.M is

(a) 10

(b) 12

(c) 14

(d) 16

(iv) The 8th decile of the following observation is 29, 18, 15, 30, 42, 35, 34, 28, 45, 34

(a) 44.6

(b) 40.6

(c) 34.15

(d) 38.15

(v) The slope of regression line of x on y is

(a) b_{yx}

(b) b_{xy}

(c) $1/b_{xy}$

(d) $1/b_{yx}$

(vi) If the two regression lines in a bivariate distribution are x + 9y = 7 and y + 4x = 16, then $S_x : S_y$ is

(a) 3 : 2

(b) 2 : 3

(c) 9 : 4

(d) 4 : 9

(vii) If the rank correlation coefficient between two variables is 0.6 and the sum of squares of the differences in ranks is 66, then number of observations is

(a) 10

(b) 9

(c) 8

(d) 11

(viii) Co-efficient of correlation between two variables x and y is 0.8 and their covariance is 20. If the variance of X series is 16, the standard deviation of y series is

(a) 32

(b) 12

(c) -6.25

(d) 6.25

(ix) A card is drawn at random from a well-shuffled pack of 52 cards, what is the probability that it is a heart or a queen

(a) 1/4

(b) 1/13

(c) 4/13

(d) none of these

(x) The letters of the word SUCCESS are to be arranged at random. What is the probability that the vowels occur at even places.

(a) 2/5

(b) 1/5

(c) 1/7

(d) 1/6

(xi) For a symmetric distribution

(a) Mean < median < mode

(b) mean ≠ median ≠ mode

(c) mean > median > mode

(d) mean = median = mode

(xii) When all the observations occur with equal frequency then which one of the following does not exist?

(a) median

(b) mode

(c) mean

(d) none

Answer:

(i) (c) one

(ii) (d) both (a) and (c)

(iii) (b) 12

(iv) (b) 40.6

(v) (c) $1/b_{xy}$

(vi) (a) 3 : 2

(vii) (a) 10

(viii) (d) 6.25

(ix) (c) 4/13

(x) (c) 1/7

(xi) (d) mean = median = mode

(xii) (b) mode

(b) State whether the following statements are True (or) False. [12×1= 12]

(i) Measures of central tendency are called averages of the 1st order

(ii) The algebraic sum of deviations of a set of observations from their AM is Negative

(iii) Quartile deviation for the data 1,3,4,5,6,6,10 is 1.5

(iv) The most commonly used measure of dispersion is standard deviation

(v) Coefficient of standard deviation is equal to AM/Standard deviation

(vi) Spearman's method is devised by Prof. Charles Edward spearman

(vii) If the plotted points in a scatter diagram lie from upper left to lower right, then the correlation is Zero.

(viii) The line X = a + by represents the regression equation of Y on X

(ix) If p : q are the odds in favour of an event, then the probability of that event is p/q

(x) Mathematical Average is called Geometric means

(xi) A distribution with two modes is called Bimodal

(xii) If coefficient of skewness is negative then $Q_3 + Q_1 > 2Q_2$

Answer:

(i) True

(ii) False

(iii) True

(iv) True

(v) False

(vi) True

(vii) False

(viii) False

(ix) False

(x) False

(xi) True

(xii) True

PART – B

4 Questions to be answered out of 6 questions [6×4=24

9. Describe the Limitations of Statistics. [6]

Answer:

Statistics and its techniques are widely used in every branch of knowledge. W.I. King rightly says: "Science of statistics is the most useful servant, but only of great value to those who understand its proper use". The scope of statistics is very wide and it has great utility; but these are restricted by its limitations. Following are the important limitations of statistics:

1. **Statistics does not deal with individual item**: King says, "Statistics from the very nature of the subject cannot and never will be able to take into account individual cases". Statistics proves inadequate, where one wants to study individual cases. Thus, it fails to reveal the true position.
2. **Statistics deals with quantitative data**: According to Prof. Horace Secrist, "Some phenomenon cannot be quantitatively measured; honesty, resourcefulness, integrity, goodwill, all important in industry as well as in life, are generally not susceptible to direct statistical measurement".
3. **Statistical laws are true only on averages**. According to W.I. King, "Statistics largely deals with averages and these may be made up of individual items radically different from each other". Statistics are the means and not a solution to a problem.
4. **Statistics does not reveal the entire story**: According to Marshall, "Statistics are the straws, out of which, I like every other economist to have to make bricks. "Croxton says: "It must not be assumed that statistical method is the only method or use in research; neither should this method be considered the best attack for every problem".
5. **Statistics is liable to be misused**: According to Bowley, "Statistics only furnishes a tool though imperfect, which is dangerous in the hands of those who do not know its use and deficiencies". W.I. King states, "Statistics are like clay of which you can make a God or Devil as you please". He remarks, "Science of Statistics is the useful servant, but only of great value to those who understand its proper use".
6. **Statical data should be uniform and homogeneous**

10. Calculate the geometric mean of the following figures by direct and by Short-cut method. [6]

5, 10, 192, 14374, 20498, 120674, 15491

Answer:

Calculation of the Geometric Mean.

Size of items (x)	**Logarithms (Log x)**	**Deviation from assumed log mean (3.0000) (d)**
5	0.6990	-2.301
10	1.0000	-2.000
192	2.2833	-.7167
14374	4.1574	1.1574
20498	4.3118	1.3118
120674	5.0816	2.0816
15491	4.1903	1.1903
n = 7	Σ Log x = 21.7234	Σd 0.7234

Direct Method

$$\text{Geometric mean} = \text{Anti-Log}\left[\frac{\text{Log a}+\text{Log b}+\ldots+\text{Log n}}{n}\right]$$

$$= \text{Anti-Log}\ \frac{\sum \text{Log x}}{n}$$

We get, $g = \text{Anti-Log}\left[\frac{21.7234}{7}\right]$

$= \text{Anti-Log } 3.1034$

$= 1269.0$

Short-cut Method

$$\text{Geometric mean} = \text{Anti-Log}\left[3+\frac{0.7234}{7}\right]$$

$= \text{Anti-Log } [3.1034]$

$= 1269.0$

11. Weekly earnings of a randon sample of 15 employees of a company are:

62, 42, 73, 80, 182, 78, 69, 103, 92, 84, 130, 58, 170, 71, 97 Find the lower quartile, upper quartile, interquartile range, range and the quartile deviation. [6]

Answer:

We first have to arrange the given figures in an array:

42, 58, 62, 69, 71, 73, 78, 80, 84, 92, 97, 103. 130, 170, 182

Total items, n = 15

Q_1 is $\frac{n+1}{4}$ th i.e. 4th item = 69.

Q_3 is $\frac{3(n+1)}{4}$ th i.e 12th item = 103.

Interquartile range is $Q_3 - Q_1$ = 103 – 69 = 34.

Range is largest item – smallest item = 182 – 42 = 140.

Quartile deviation = $\frac{Q_3 - Q_1}{2} = \frac{103-69}{2} = \frac{34}{2}$ = 17.

12. Find the co-efficient of correlation from the following data: [6]

X:	3	5	7	8	9	15	16
Y:	15	18	22	24	19	25	31

Answer:

Calculation

X	Y	$x = X - \bar{X}$	$y = Y - \bar{Y}$	x^2	y^2	xy
3	15	— 6	— 7	36	49	42
5	18	— 4	— 4	16	16	16
7	22	— 2	0	4	0	0
8	24	— 1	2	1	4	— 2
9	19	0	— 3	0	9	0
15	25	6	3	36	9	18
16	31	7	9	49	81	63
63	154	—	—	142	168	137

$\bar{X} = \frac{\Sigma X}{n} = \frac{63}{7} = 9$; $\bar{Y} = \frac{\Sigma X}{n} = \frac{154}{7} = 22$

$\therefore \quad r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \times \Sigma y^2}} = \frac{137}{\sqrt{142 \times 168}} = \frac{137}{\sqrt{23856}} = \frac{137}{154.45} = 0.887.$

13. From the data given below find (i) the two regression equtions (ii) the coefficent of coordination between marks in Economics and Statistics, (iii) the most likely marks in statistics when the marks in Economics is 30 : [6]

Marks in Economics (x):	25	28	35	32	31	36	29	38
Marks in Statistics (y):	43	46	49	41	36	32	31	30

Answer:

Calculate of regression equations

X	y	u (= x–31)	v = (y–41)	u^2	v^2	uv
25	43	-6	2	36	4	-12
28	46	-3	5	9	25	-15
35	49	4	8	16	64	32
32	41	1	0	1	0	0
31	36	0	-5	0	25	0
36	32	5	-9	25	81	-45
29	31	-2	-10	4	100	20
38	30	7	-11	49	121	-77
Total		**6**	**-20**	**140**	**420**	**-97**

$\bar{x} = 31 + \frac{6}{8} = 31 + 0.75 = 31.75$; $\bar{y} = 41 + \frac{(-20)}{8} = 41 - 2.5 = 38.5.$

$b_{uv} = \frac{n(\Sigma uv) - \Sigma u \Sigma v}{n(\Sigma v^2) - (\Sigma v)^2} = \frac{8 \times (-97) - 6 \times (-20)}{8 \times 420 - (-20)^2} = \frac{-776 + 120}{3360 - 400} = \frac{-656}{2960} = -0.22$

$b_{vu} = \frac{n(\Sigma uv) - \Sigma u \Sigma v}{n(\Sigma u^2) - (\Sigma u)^2} = \frac{8 \times (-97) - 6 \times (-20)}{8 \times 140 - 6^2} = \frac{-656}{1084} = -0.605 = -0.61$

As regression coefficients are independent of origin, so

$b_{xy} = b_{uv} = -0.22$ and $b_{yx} = b_{vu} = -0.61$

Regression eqn. of x on y : $x - \bar{x} = b_{xy}(y - \bar{y})$ or, $x - 31.75 = -0.22\ (y - 38.5)$

or, $x = -0.22y + 40.22$

Regression eqn. of y on x : $y - \bar{y} = b_{yx}(x - \bar{x})$ or $y - 38.5 = -0.61\ (x - 31.75)$

or, $y = -0.61x + 57.87.$

(ii) $r = \sqrt{(-.22) \times (-.61)} = -0.366$

(iii) y (likely marks in stat.) = – 0.61 × 30 + 57.87 = – 18.3 + 57.87 = 39.57 = 40.

14. A can hit a target 3 times in 5 shots, B 2 times in 5 shots, C 3 times in 4 shots. They fire a volley. What is the probability that 2 shots hit ? [6]

Answer:

The term fire a volley means all three persons try simultaneously to hit the target. Two shots hit the target may occur in any one of the following ways :

(i) A and B hit and C fails to hit

(ii) A and C hit and B fails to hit

(iii) B and C hit and A fails to hit

Let A : event A hits : B : event B hits ; C : event C hits.

(i) In this case events A, B, C are indepdent

$\therefore\ P(A \cap B \cap \bar{C}) = P(A) \cdot P(B) \cdot P(\bar{C})$

$= \frac{3}{5} \times \frac{2}{5} \times \frac{1}{4} = \frac{6}{100}$ $\left[\text{as } P(C) = \frac{3}{4} P, (\bar{C}) = 1 - \frac{3}{4} = \frac{1}{4}\right]$

(ii) Similarly, $P(A \cap C \cap \bar{B}) = P(A) \cdot P((C) \cdot P(\bar{B}) = \frac{3}{5} \times \frac{3}{4} \times \frac{3}{5} = \frac{27}{100}$

$P(\bar{B}) = 1 - P(B) = 1 - \frac{2}{5} = \frac{3}{5}$

(iii) $P(B \cap C \cap \bar{A}) = P(B) \cdot P(C) \cdot P(\bar{A})$

$= \frac{2}{5} \times \frac{3}{4} \times \frac{2}{5} = \frac{12}{100}$, $P(A) = 1 - P(\bar{A}) = 1 - \frac{3}{5} = \frac{2}{5}$.

Since (i), (ii) and (iii) are mutually exclusive, exhaustive and equally likely, so the probability that 2 shots hit the target

$= \frac{6}{100} + \frac{27}{100} + \frac{12}{100} = \frac{45}{100}$ i.e 45%