Paper 4 - Fundamentals of Business Mathematics and Statistics

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Time Allowed: 3 Hours **Full Marks: 100**

The figures in the margin on the right side indicate full marks.

This question paper has two sections.

Both the sections are to be answered subject to instructions given against each.

Section – A

I. (a) Choose the correct answer **(9 × 2 = 18)**

(1) The number to be added to each term of the ratio 3 : 7 to make it 1: 2 is
(a) 2, (b) 1, (c) 3, (d) none of these

(2) A person deposits ₹ 2,000 @ 6% p.a. simple interest for 3 years. The amount he will get back after 3 years is
(a) 2300 (b) 2400 (c) 2360 (d) 2350

(3) If one roots of the equation $x^2 - 3x + m = 0$ exceeds the other by 5 then the value of M is equal to _____
(a) -6 (b) -4 (c) 12 (d) 18

(4) If $^{n}p_3 = 120$ then n = ___
(a) 8 (b) 4 (c) 6 (d) None of these

(5) If $\log_{10}^{2} = 0.3010$ $\log_{2}^{10} =$____
(a) 0.3322 (b) 3.2320 (c) 3.3222 (d) 5

(6) If $^{r}c_{12} = {}^{r}c_8$ find $^{22}c_r$
(a) 213 (b) 321 (c) 231 (d) None of these

(7) The number of ways in which letters of the word Monday be arranged beginning with the letter O and ending the letter Y is
(a) 120 (b) 24 (c) 96 (d) None of these

(8) Some money is distributed between A and B in the ratio 2:3. If A receives Rs. 72, then B receives:
(a) Rs. 90 (b) Rs. 144 (c) Rs. 108 (d) None of these

(9) The sum of the first 5 and first 10 terms of a G. P. are respectively 16 and 3904. Find the common ratio.
(a) 2 (b) 3 (c) 4 (d) 5

I. (b) State whether the following statements are true or false (6 × 1 = 6)

(1) The average of 50 numbers is 38. If two numbers, namely 45 and 55 are discarded, the average of the remaining numbers is 30. ( )

(2) If the terms -1 + 2x, 5, 5+x are is an A.P. then x is 4 ( )

(3) The logarithm of one to any base is zero ( )

(4) If $^nC_n = 1$ then 0! = 1 ( )

(5) The number of different number of 6^{th} digits (without repetition) can be formed form the digits 3,1,7,0,9,5 is 120 ( )

(6) The degree of the equation $3x^5 + xyz^2 + y^3$ is 3 ( )

Answer: I (a)

(1) $\frac{3+x}{7+x} = \frac{1}{2} \Rightarrow x = 1$ (Option b)

(2) {2000 × (6/100) × 3 = 360 + 2000} (Option c)

(3) $\because x^2 - 3x + m = 0$

Let the roots be α, $\alpha + 5$

$\because \alpha + (\alpha + 5) = 3$

$2\alpha = -2$

$\alpha = -1$

$\therefore$ the roots be -1, 4

$\therefore$ Product of roots = M = -4 (Option b)

(4) $\because {}^nP_3 = 120$ or $\frac{\lfloor n}{\lfloor n-3} = 120$

$\Rightarrow n(n-1)(n-2) = 120 = 6 \times 5 \times 4$

$\therefore n = 4$ (Option c)

(5) $\log_2 10 = \frac{1}{\log_{10} 2}$ $\frac{1}{0.3010} = 3.3222$ (Option c)

(6) $\because {}^rC_{12} = {}^rC_8 \Rightarrow r = 12 + 8 = 20.$

$\because {}^{22}C_y = {}^{22}C_{20} = \frac{\lfloor 22}{\lfloor 20 \lfloor 2} = \frac{22 \times 21}{2} = 21 \times 11 = 231$

(Option c)

(7) (Option b)

(8) A : B = 2 : 3

B = (72/2) × 3 = 108 (Option c)

(9) (Option b)

Answer: I (b)

(1) The average of 50 numbers is 38. If two numbers, namely 45 and 55 are discarded, the average of the remaining numbers is 30. (F)

(2) If the terms -1 + 2x, 5, 5+x are is an A.P. then x is 4 (F)

(3) The logarithm of one to any base is zero (T)

(4) If ${}^{n}C_{n} = 1$ then 0! = 1 (T)

(5) The number of different number of 6^{th} digits (without repetition) can be formed form the digits 3,1,7,0,9,5 is 120 (F)

(6) The degree of the equation $3x^5 + xyz^2 + y^3$ is 10 (F)

II. Answer any four questions. Each question carries 4 marks (4 × 4 = 16)

(1) The ratio of present age of mother to her daughter is 5: 3. Ten years hence the ratio would be 3 : 2. Find their present ages.

(2) What sum of money will yield ₹ 1,407 as interest in 1½ year at 14% p.a. simple interest?

(3) Insert 4 arithmetic means between 4 and 324.

(4) If $\frac{\log x}{y^2+z^2+yz} = \frac{\log y}{z^2+x^2+zx} = \frac{\log z}{x^2+y^2+xy}$

Show that $x^{y-z}\, y^{z-x}\, z^{x-y} = 1$

(5) From a company of 15 men, how many selections of 9 men can be made so as to exclude 3 particular men?

(6) If a, 4, b are in AP and a, 2, b are in G.P., then prove that 1/a + 1/b = 2.

Answer: II

(1) Let present age of mother be 5x and that of her daughter be 3x years.
10 years hence age of mother will be (5x + 10) years and that of daughter be (3x + 10) years.
By question $\frac{5x+10}{3x+10} = \frac{3}{2}$ or , 2(5x + 10) = 3(3x + 10) or, 10x + 20 = 9x + 30 or, x = 10
∴ Reqd. ages are 5 × 10 = 50 years and 3 × 10 = 30 years.

(2) What sum of money will yield ₹ 1,407 as interest in 1 year at 14% p.a. simple interest.
Here S.I = 1407, n = 1.5, I = 0.14, P = ?
S.I = P. i.n or, 1407 = p x 0.14 x1.5
Or, $p = \frac{1407}{0.14\times1.5} = \frac{1407}{0.21} = 6700$
Required amount = ₹ 6,700%

(3) Let a = 4, b = 324

$d = \left(\frac{b}{a}\right)^{\frac{1}{x+1}} = \left(\frac{239}{4}\right)^{\frac{1}{5}} = (81)^{\frac{1}{3}}$

$\because tn = b$

$\Rightarrow a + (n+1)\,d = b$

$d = \frac{b-a}{n+1} = \frac{324 - 4}{5} = \frac{320}{5}$ 64

t_1, = 68, t_2 = 132, t_3 = 196, t_4 = 260

(4) $\frac{\log x}{y^2 + z^2 + yz} = \frac{\log y}{z^2 + x^2 + zx} = \frac{\log z}{x^2 + y^2 + xy} = k$ (say)

Or log x = k ($y^2 + z^2 + yz$), log y = k ($z^2 + x^2 + zx$), log z = k ($x^2 + y^2 + xy$) ……(i)

To show $x^{y-z}\, y^{z-x}\, z^{x-y} = 1$, taking logarithm both sides

Log ($x^{y-z} . y^{z-x} . z^{x-y}$) = log 1 = 0 i. e. to show

(y – z) log x + (z – x) log y + (x- y) log z = 0

L. H. S. = (y – z) . k. ($y^2 + z^2 + yz$) + (z – x). k. ($z^2 + x^2 + zx$) + (x – y). k. ($x^2 + y^2 + xy$)

= k ($y^3 – z^3 – x^3 + x^3 – y^3$) = k. 0 = 0, hence proved.

(5) Excluding 3 particular men in each case, we are to select 9 men out of (15-3) men. Hence the number of selection is equal to the number of combination of 12 men taken 9 at a time which is equal to

$^{12}C_9 = \frac{12!}{9!3!} = 220$

(6) If a,4 b are in A.P. and a, 2, b are in G.P., then Prove that $\frac{1}{a} + \frac{1}{b} = 2$

$\therefore$ a, 4, b are in A.P., then 4-a = b-4

Or, a+b = 8 …..(1)

Again a, 2, b are in G.P., then $\frac{2}{a} = \frac{b}{2}$

Or, ab = 4 ……(2)

Dividing (1) by (2), we get,

$\frac{a}{ab} + \frac{b}{ab} = \frac{8}{4}$

Or, $\frac{1}{b} + \frac{1}{a} = 2$

Or, $\frac{1}{a} + \frac{1}{b} = 2$

Section – B

III. (a) Choose the correct answer **(12 × 2 = 24)**

(1) For the observations 6, 4, 1, 6, 5, 10, 4, 8 range is
(a) 10 (b) 9 (c) 8 (d) None

(2) Given $\sum_{i=1}^{n}(x_i - 4) = 72$ and $\sum_{i=1}^{n}(x_i - 7) = 3$. Then arithmetic mean of x is
(a) 68.8 (b) 6.88 (c) 0.688 (d) none of these

(3) Harmonic mean is used for calculating
(a) Average Growth Rate of variables (b) Average speed of journey
(c) Average rate of increase in net worth of a company (d) All the above 1 to 3

(4) $x = \frac{31}{6} - \frac{y}{6}$ is the regression equation of
(b) y on x (b) x on y (c) both (d) none

(5) The mean of first 10 even number is
(a) 5.5 (b) 55 (c) 11 (d) none of these

(6) For two positive observations x^1 and x^2 which one of the following is true?
(AM) (HM) = $(GM)^2$ (b) (AM) (GM) = $(HM)^2$
(c) (GM) (HM) = $(AM)^2$ (d) None of above

(7) If y = a + bx, then what is the co-efficient of correlation between x and y?
(a) 1 (b) -1 (c) 1 or -1 according as b > 0 or b < 0 (d) None of these

(8) The lower & upper quartiles are used to define
(a) Standard deviation (b) Quartile Deviation (c) Both
(d) None

(9) If an unbiased coin is tossed twice, the probability of obtaily of obtaining at least one tail is
(a) 0.25 (b) 0.50 (c) 0.75 (d) 1.00

(10) Difference between the maximum & minimum value of a given data is called –
(a) Width (b) Size (c) Range (d) Class

(11) Two dice are thrown together. The probability that 'the event the difference of nos. shown is 2' is
(a) 2/9 (b) 5/9 (c) 4/9 (d) 7/9

(12) If an unbiased coin is tossed twice, the probability of obtaining at least one tail is
(a) 0.25 (b) 0.50 (c) 0.75 (d) 1.00

III. (b) State whether the following statements are true or false **(12 × 1 = 12)**

(1) There is no difference between co-efficient of variation and variance ( )

(2) Sum of probability of an event A and its complements is 1 ( )

(3) The slope of the regression line of y on x is b_{yx} ( )

(4) If events are mutually exclusive then their probabilities are less than one ( )

(5) In a moderately asymmetrical distribution A.M. < G.M. < H.M. ( )

(6) Median can never be equal to mean in a skewed distribution ( )

(7) The sum of individual observations from mean is one ( )

(8) If x and y satisfy the relationship y = -5 +7x, the value of r is zero ( )

(9) In a normal distribution SD < MD < QD ( )

(10) Mode is the value that has maximum frequency ( )

(11) In the line $y = 19 - \frac{5x}{2}$, b_{yx} is equal to -5/2 ( )

(12) Sum of all probabilities is equal to one ( )

Answer: III (a)

(1) (b)

(2) (a)

(3) (b)

(4) (b)

(5) (c)

(6) (a)

(7) (c)

(8) (b)

(9) (c)

(10) (c)

(11) (a)

(12) (c)

Answer: III (b)

(1) (F)

(2) (T)

(3) (T)

(4) (F)

(5) (T)

(6) (T)

(7) (F)

(8) (F)

(9) (F)

(10) (T)

(11) (T)

(12) (T)

IV. Answer any four questions. Each question carries 6 marks (4 × 6 = 24)

(1) Draw the histogram of the following data and comment on the shape of the distribution:

Wages (in ₹)	**:**	**50-59**	**60-69**	**70- 79**	**80-89**	**90-99**
No. of employees	**:**	**8**	**10**	**16**	**12**	**7**

(2) The mean and standard deviation of the marks obtained by the groups of the students consisting of 50 each are given below:

Group	Mean	S.D.
A	60	8
B	55	7

Calculate the mean and standard deviation of the marks obtained by all 100 students.

(3) The marks obtained by 6 students were 24, 12, 16, 11, 40, 42. Find the Range. If the highest mark is omitted, find the percentage change in the range.

(4) Compute rank correlation from the following table

X	**415**	**434**	**420**	**430**	**424**	**428**
Y	**330**	**332**	**328**	**331**	**327**	**325**

(5) The means of samples of sizes 50 and 75 are 60 and x respectively. If the mean of the combined group is 54, find x.

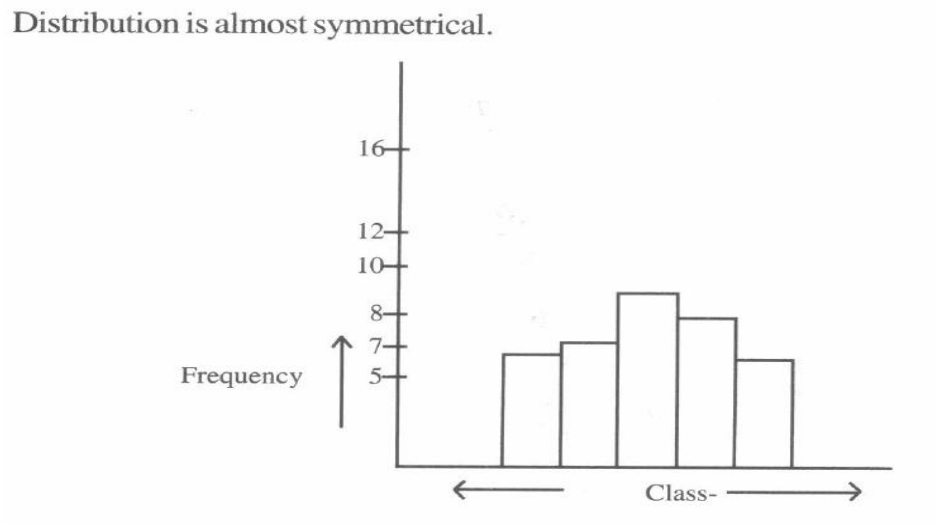
(6) A bag contains 7 red balls and 5 white balls. 4 balls are drawn at random. What is the probability that (i) all of them are red; (ii) two of them are red and two white?

Answer: IV

(1)

Class- boundaries	:	49.5- 59.5	59.5 – 69.5	69.5 – 79.5	79.5 – 89.5	89.5-99.5
Frequency	:	8	10	16	12	7

HISTOGARM:



(2) Here $n_1 = 50$, $n_2 = 50$, $\bar{X}_1 = 60$, $\bar{X}_2 = 55$

So, $\bar{X} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \dfrac{50\times60 + 50\times55}{100} = \dfrac{300+2750}{100} = \dfrac{5750}{100} = 57.50$

Now $d_1 = \bar{X}_1 - \bar{X} = 60 - 57.5 = 2.5$, $d_2 = \bar{X}_2 - \bar{X} = 55 - 57.5 = -2.5$

$S_1^2 = 64$, $S_2^2 = 49$

Hence, $S^2 = \dfrac{n_1(S_1^2 + d_1^2) + n_2(S_2^2 + d_2^2)}{n_1 + n_2}$

$= \dfrac{50[64 + 6.25 + 49 + 6.25]}{100} = \dfrac{50\times125.50}{100} = 62.75$

$S = \sqrt{62.75} = 7.92$

Hence, Mean $\bar{X} = 57.5$ and std deviation (s) = 7.92.

(3) The marks obtained by 6 students were 24, 12, 16, 11, 40, 42. Find the Range. If the highest mark is omitted, find the percentage change in the range.
Here maximum mark = 42, minimum mark = 11.

$\therefore$ Range = 42 - 11 = 31 marks

If again the highest mark 42 is omitted, then amongst the remaining. Maximum mark is 40. So, i (revised) = 40 - 11 =29 marks.

Change in range = 31 - 29 = 2 marks.

$\therefore$ Reqd. percentage change = 2 ÷ 31 × 100 = 6.45%

Note: Range and other obsolute measures of dispersion are to be expressed in the same unit in which observations are expressed.

For grouped frequency distribution:

In this case range is calculated by subtracting the lower limit of the lowest class interval from the upper limit of the highest.

(4)

X	R_1	Y	R_2	$(R_1 - R_2) = D$	D^2
415	6	330	3	3	9
434	1	332	1	0	0
420	5	328	4	1	1
430	2	331	2	0	0
424	4	327	5	-1	1
428	3	325	6	-3	9

$$r_k = 1 - \frac{6\Sigma D^2}{N(N^2 - 1)}$$

$$= 1 - \frac{1(20)}{6(6^2 - 1)} = 1 - \frac{120}{210} = \frac{210 - 120}{210} = \frac{90}{210} = \frac{3}{7} = 0.429$$

(5) We have $\overline{x_{12}} = \frac{n_1\overline{x_1} + n_1\overline{x_1}}{n_1 + n_2}$ or, $54 = \frac{50 \times 60 \times 75x}{50 + 75}$ or, $54 = \frac{3000 \times 75x}{125}$

or, $3000 + 75x = 6750$ or, $75x = 3750$ or, $x = 50$.

(6) (i) Favourable cases ${}^7C_4$, Exhaustive cases ${}^{12}C_4$.

Probability = $\frac{{}^7C_4}{{}^{12}C_4} = \frac{105}{495} = \frac{7}{33}$

(ii) Favourable cases = ${}^7C_2 \times {}^5C_2$

Exhaustive cases = ${}^{12}C_4$

Probability = $\frac{{}^7C_2 \times {}^5C_2}{{}^{12}C_4} = \frac{12 \times 10}{495} = \frac{14}{33}$