# Paper- 4: FUNDAMENTALS OF BUSINESS MATHEMATICS AND STATISTICS

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**Full Marks: 100** **Time Allowed: 3 Hours**

**Section – A**

**I. Answer any TWO questions. Each question carries 5 marks [2×5 = 10]**

1. Let 'x' be the original no. of labourers in a factory and ₹ 'y' be the original average wages of labourers.

∴ Original amount of wages = ₹ xy

After reduction, No. of Labourers = $\frac{11}{15}x$

Amount of wages after reduction in the no. of labourers and increment in their wages = $\frac{11}{15}x \times \frac{25}{22}y$ = ₹ $\frac{5}{6}xy$.

∴ Required Ratio = $\frac{\text{Original wages}}{\text{Changed wages}} = \frac{xy}{\frac{5}{6}xy}$ = = 6 : 5 (decreased)

2. Given $\log_a bc = x$, $\log_b ca = y$ and $\log_c (ab) = z$

Now $x+1 = \log_a (bc) + \log_a a = \log_a abc$

Similarly $y+1 = \log_b abc$ , $z+1 = \log_c (abc)$

Now, L.H.S. $= \frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$

$= \frac{1}{\log_a abc} + \frac{1}{\log_b (abc)} + \frac{1}{\log_c (abc)}$

$= \log_{abc} a + \log_{abc} b + \log_{abc} c$

$= \log_{abc} (abc)$

$= 1$ = R.H.S.

3. Given total cost of steel plant is $c(x) = \frac{1}{3}x^3 - 7x^2 + 11x + 50$

Marginal cost (MC) = $\frac{dc}{dx}$

$= \frac{1}{3}(3x^2) - 7(2x) + 11$

$= x^2 - 14x + 11$ (say y)

In order to that the marginal cost is to be Minimum its derivative is zero and 2nd derivative must be positive.

∴ $\frac{dy}{dx} = 0$

⇨ $2x - 14 = 0$

⇨ X = 7 tons / week

Again $\frac{d^2y}{dx^2} = 2 > 0$ (which is +ve)

∴ Marginal cost is Minimum at x = 7 tons/week

**II. Answer any TWO questions. Each question carries 3marks [2 × 3 = 6]**

4. Let the sum be ₹ 'p'

∴ i = 5% = 0.05, n = 2yrs.

c.i. = $p\{(1+i)^n - 1\} = p\{(1.05)^2 - 1\}$

$= p\{1.1025 - 1\}$
$= p\{0.1025\}$

S.I. $= \frac{prt}{100} = P(0.05)(2)$
$= P\ (0.10)$
$\because$ C.I. – S.I. = 6.90
$\Rightarrow P(0.1025) - P(0.10) = 6.90$
$\Rightarrow P(0.0025) = 6.90$
$\therefore P = \frac{6.90}{0.0025} = ₹\ 2760$
$\therefore$ Required sum = ₹ 2760.

5. Let $(1.234)^a = (0.1234)^b = 10^c = k$ (say)

| | | |
|---|---|---|
| $\therefore (1.234)^a = k$<br>$a = \log_{1.234} k$<br>$\therefore \frac{1}{a} = \log_k (1.234)$ | $\therefore (1.234)^b = k$<br>$\Rightarrow b = \log_{0.1234} k$<br>$\Rightarrow \frac{1}{b} = \log_k (0.1234)$ | $\therefore 10^c = k$<br>$c = \log_{10} k$<br>$\frac{1}{c} = \log_k 10$ |

Now L.H.S. $= \frac{1}{a} - \frac{1}{c}$
$= \log_k (1.234) - \log_k 10$
$= \log_k (\frac{1.234}{10})$
$= \log_k (0.1234)$
$= \frac{1}{b}$
= R.H.S.

6. Given a function f(x) is defined as follows
f (x) = 2x – 1 $\quad x < 3$
= k $\quad x = 3$
= 8 – x $\quad x > 3$

$\because$ f(x) is continuous at x = 3
i.e., $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3)$
$\Rightarrow \lim_{x \to 3^-} (2x - 1) = \lim_{x \to 3^+} (8 - x) = k$
$\Rightarrow \lim_{n \to 0} \{2(3 - h) - 1\} = \lim_{n \to 0} 8 - (3 + n) = k$
$\Rightarrow 5 = 5 = k.$

**III. Choose the correct answer** **[5 × 1 = 5]**

7. Answer : (a)
Let the fraction be x
$\therefore x : \frac{1}{27} = \frac{3}{11} : \frac{5}{9}$

$\Rightarrow \frac{x}{\frac{1}{27}} = \frac{\frac{3}{11}}{\frac{5}{9}}$

⇨ $\cancel{27}x = \frac{3}{11} \times \frac{9}{5} = \frac{\cancel{27}}{55}$

$x = \frac{1}{55}$

8. Answer : (a)

Given $A \propto B^2$

⇨ $A = KB^2$

∵ When A = 4 then B = 4

∴ $4 = k(4)^2$

$K = \frac{4}{16} = \frac{1}{4}$

∵ A = 3

$3 = \frac{1}{4} B^2$

$B^2 = 12$

9. Answer : (d)

Let the required number be x

∵ $\cancel{3}x = \frac{\cancel{3}}{5}x^2$

$x^2 = 5x$

$x^2 - 5x = 0$

⇨ $x(x - 5) = 0$

⇨ $x = 0$ (or ) $x = 5$

10. Answer : (c)

∵ $y = \log(2x + 5)$

$\frac{dy}{dx} = \frac{1}{2x+5} \times 2 = \frac{2}{2x+5}$

11. Answer : No. option

$\int_2^4 3dx = 3\int_2^4 dx = 3(x) \mid_2^4$

$= 3(4 - 2)$

$= 6$

**IV. Choose the correct answer** **[5 × 1 = 5]**

12. Let the sum be ₹ p

∵ $\cancel{x} = p \frac{x}{100} \times \cancel{x}$

$p = (\frac{100}{x})$ (₹)

13. Let 'n' be the total no. of personal in a room.

∴ ${}^{n}c_{2} = 66$

⇨ $\frac{\lfloor n}{\lfloor n-2 \ \lfloor 2} = 66$

$\Rightarrow \frac{n(n-1)(n-2)!}{(n-2)!} = 132$

$\Rightarrow n(n-1) = 12 \times 11$

$\therefore n = 12$ persons

14. $\because b^2 - 4ac = 0$

$\Rightarrow 6^2 - 4(1)(c) = 0$

$\Rightarrow 4c = 36$

$\Rightarrow C = \frac{36}{4} = 9$

15. Given $f(x) = 2x^2 - 5x + 4$ and

$2f(x) = f(2x)$

$\Rightarrow 2[2x^2 - 5x + 4] = 2(2x)^2 - 5(2x) + 4$

$\Rightarrow 4x^2 - 10x + 8 = 2(4x^2) - 10x + 4$

$\Rightarrow 4x^2 + 8 = 8x^2 + 4$

$\Rightarrow 4x^2 = 4$

$\Rightarrow x^2 = 1$

$\Rightarrow x = \pm 1$

16. $\lim_{n \to D} \frac{1}{7} + \frac{1}{7^2} + \ldots\ldots + \frac{1}{7^n}$

$\Rightarrow = \frac{a}{1-r} = \frac{\frac{1}{7}}{1-\frac{1}{7}}$ $\quad (\because S_D = \frac{a}{1-r})$

$= \frac{\frac{1}{7}}{\frac{6}{7}}$

$= \frac{1}{6}$

**V. State whether the following statements are true or false** **[5 × 1 = 5]**

17. Answer : True

$\because$ 15% of x = 20% of y

$\Rightarrow \frac{15}{100} x = \frac{20}{100} y$

$\Rightarrow 15x = 20y$

$\therefore 3x = 4y$

$\Rightarrow \frac{x}{y} = \frac{4}{3} = 4 - 3$

18. True

19. False

We know that No. of diagonals of polygon is ${}^{n}C_2 - n$

$\therefore {}^{n}C_2 - n = 44$ (Given)

$\Rightarrow \frac{\lfloor n}{\lfloor n-2 \; 2!} - n = 44$

$\Rightarrow \frac{n(n-1)\lfloor n-2}{\lfloor n-2 \; 2} - n = 44$

⇨ $n^2 - n - 2n = 88$
⇨ $n^2 - 3n - 88 = 0$
∴ $n^2 - 11n + 8n - 88 = 0$
⇨ $n(n-11) + 8(n - 11) = 0$
⇨ $(n-11)(n+8) = 0$
∴ $n = 11$ (or) $n = -8$ (not possible)

20. False

$$\lim_{x \to 3} \frac{x^2 - 4}{x + 1} = \frac{9 - 4}{4} = \frac{5}{4}$$

21. False

∵ Demand function of product $D = 12 - x^2$
∴ Total Revenue = Dx
(R) $= (12 - x^2)\, x$
$= 12x - x^3$
∴ Marginal Revenue (MR) $= \frac{dR}{dx}$
$= 12 - 3x^2$

**VI. Match the following** **[5 × 1 = 5]**

| | | |
|---|---|---|
| 22. | $\frac{2}{3}$ years | (D) |
| 23. | ∵ $2x + 3y - 5 = 0$ and $kx - 6y - 8$ have<br>Unique solution<br>∴ $\frac{2}{x} = \frac{3}{-6} = \frac{-5}{-8}$<br>$\frac{2}{k} = \frac{3}{-6} = \frac{-1}{2}$<br>$k = -4$ | (C) |
| 24. | ∵ A = B<br>$= \begin{bmatrix} x-2 & 4 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 3 & 5 \end{bmatrix}$<br>∴ $x - 2 = -1$<br>$x = 2 - 1$<br>$x = 1$ | (A) |
| 25. | $\lim_{x \to o} \frac{1 - \sqrt{1-x^2}}{x^2}$<br>$= \lim_{x \to o} \frac{1-\sqrt{1-x^2}}{x^2} \times \frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}}$<br>$= \lim_{x \to o} \frac{1-(1-x^2)}{x^2(1+\sqrt{1-x^2})}$ | (B) |

| | | |
|---|---|---|
| | $= \underset{x\to o}{Lt} \frac{x^2}{x^2(1+\sqrt{1-x^2})}$<br>$= \frac{1}{1+1} = \frac{1}{2}$ | |
| 26. | $\int_1^2 xe^x \, dx$<br>$= x^{ex} \Big\|_1^2 - \int_1^2 1.e^x \, dx$<br>$= 2e^2 - e - e^x \Big\|_1^2$<br>$= 2e^2 - e - (e^2 - e)$<br>$= 2e^2 - e - e^2 - e$<br>$= e^2$ | (E) |

**VIII. Answer the following in one or two steps** **[4 × 1 = 4]**

27. ∵ A = {1, 2, 3} and B = { 1, 2, 3, 4}

A Δ B = (A – B) ∪ (B – A)

= { } ∪ { 4}

= {4 }

28. √5 is a rational number (OR) √5 is an irrational number.

29. ∵ $A=\begin{pmatrix}1 & 2\\ 9 & 4\end{pmatrix}$ ; $B\begin{pmatrix}3 & 12\\ 13 & 52\end{pmatrix}$

Let x = $\begin{pmatrix}a & b\\ c & d\end{pmatrix}$

Given condition, Ax = B

$$= \begin{pmatrix}1 & 2\\ 9 & 4\end{pmatrix} \begin{pmatrix}a & b\\ c & d\end{pmatrix} = \begin{pmatrix}3 & 12\\ 13 & 52\end{pmatrix}$$

$$= \begin{pmatrix}a+2c & b+2d\\ 9a+4c & 9b+4d\end{pmatrix} = \begin{pmatrix}3 & 12\\ 13 & 52\end{pmatrix}$$

$$\begin{array}{l} 2a + 4c = 6 \\ 9a + 4c = 13 \\ - \quad - \quad - \\ \hline -7a = -7 \end{array}$$

$\therefore$

$a = 1$

$\therefore$ $1 + 2c = 3$

$2c = 2$

$c = 1$

Similarly,

$2b + 4d = 24$

$9b + 4d = 52$

$- \quad - \quad -$

$-7b = -28$

$b = 4$

$\therefore 4 + 2d = 12$

$2d = 8$

$d = 4$

$\therefore$ The matrix $x = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 1 & 4 \end{bmatrix}$

30. Given in equality is $-2 (m - 3) < 5 (m + 1) - 12$

$= -2m + 6 < 5m - 7$

$= -7m < -13$

$= 7m < 13$

$= m < \frac{13}{7}$

**Section – B**

**1. Answer any Nine questions of the following** **[9× 2 = 18]**

(i) (d)

(ii) (c)

(iii) (c)

Given $N_1 = 40$; $\bar{x}_1 = 5200$

$N_2 = 60$; $\bar{x}_2 = 6800$

Combined Mean $\left(\bar{x}_{12}\right) = \frac{N_1\bar{X}_1 + N_2\bar{X}_2}{N_1 + N_2}$

$= \frac{40 \times 5200 + 60 \times 6800}{40 + 60}$

$= \frac{208000 + 408000}{100} = \frac{616000}{100} = 6160.$

(iv) (d)

Let the two numbers are a, b

Arithmetic Mean = 34

⇨ $\frac{a+b}{2} = 34$

$a + b = 68$

Geometric Mean = 16

⇨ $\sqrt{a+b} = 34$

$ab = 256$

i.e., the two numbers are 64, 4.

Greater number is 64.

(v) (a)

$Z = 3M - 2\bar{x}$

$-Z = -3M + 2\bar{x}$

$\bar{x} - Z = 3\bar{x} - 3M = 3(\bar{x} - M)$

(vi) Given N = 8, $ED^2 = 50$

We know that $r_k = 1 - \frac{6\sum D^2}{N^3 - N} = 1 - \frac{6(50)}{8^3 - 8} = 1 - \frac{300}{504} = 0.40.$

(vii) (a)

(viii) (b)

We know that

$P(AUB) = P(A) + P(B) - P(A \cap B)$

A and B are independent events, then

$P(AUB) = P(A) + P(B) - P(A)\,P(B)$

$\frac{2}{3} = \frac{2}{5} + P(B) - \frac{2}{5}P(B)$

$\frac{2}{3} - \frac{2}{5} = P(B)\left(1 - \frac{2}{5}\right)$

$\frac{4}{15} = \frac{3}{5}P(B)$

$P(B) = \frac{4}{9}$

(ix) (c)

Given $\cap(s) = 25$

Let A be the event of divisible by 4

B be the event divisible by 7

$P(A) = \frac{6}{25}$ ; $P(B) = \frac{3}{25}$ ; $P(A \cap B) = 0$

$P(AUB) = \frac{6}{25} + \frac{3}{25} = \frac{9}{25} = 0.36.$

(x) (a)

We know that

$$P(x = r) = \frac{C^{-M}M^r}{r!}$$

Given

$P(x = 2) = 9\,P(x = 4) + 90\,P(x = 6)$

$$\frac{C^{-M}M^r}{2!} = 9\,\frac{C^{-M}M^r}{4!} + 90\,\frac{C^{-M}M^r}{6!}$$

$$\frac{m^2}{2} = m^2\left(\frac{\overset{3}{\cancel{9}}m^2}{\cancel{24}_8} + \frac{\cancel{90}\,m^4}{\cancel{720}_8}\right)$$

$4 = 3m^2 + m^4$

$(m^2)^2\,3m^2 - 4 = 0$

$(m^2)^2 + 4m^2 - m^2 - 4 = 0$

$m^2\,(m^2\,4)\text{-}1\,(m^2 + 4) = 0$

$(m^2 + 4)\,(m^2 - 1) = 0$

$m^1 - 1 = 0$ $\qquad$ $m^1 + 4 = 0$

$m^2 = 1$ $\qquad$ $m^2 \neq -4$

$m = \pm 1$

$\therefore m = 1$

Standard Deviation = m = 1

(xi) (c)

$P(x = 1) = P(x = 2)$

$$\frac{c^{-m}m^1}{1!} = \frac{c^{-m}m^2}{2!}$$

$$m = \frac{m^{2!}}{2}$$

$\therefore$ m = 2

$\therefore$ Mean ($\bar{x}$) = 2

(x) (b)

| Group | Weight (W) | Index Number (I) | W.I. |
|---|---|---|---|
| Food | 35 | 425 | 14875 |
| Clothing | 15 | 235 | 3525 |
| Power & Fuel | 20 | 215 | 4300 |
| Rent and Rates | 8 | 115 | 920 |
| Miscellaneous | 22 | 150 | 3300 |
| | 100 | | 26920 |

$$P_{01} = \frac{\sum W.I.}{\sum W} = \frac{26920}{100} = 269.2.$$

**2. Answer any Nine question of the following [9×2 = 18]**

**Each question carries 2 marks**

i) histogram

ii)

| Income (In ₹) | No. of Persons |
|---|---|
| 499.5 – 999.5 | 15 |
| 999.5 – 1499.5 | 28 |
| 1499.5 – 1999.5 | 36 |
| 1999.5 – 2499.5 | 7 |
| | 86 |

$\therefore$ The number of persons earning more than ₹ 1500

$= \frac{43}{86} \times 100 = 50.$

iii) Given Median (M) =

N = 7.

Given data is 5, 9, 11, 3, 4, x, 8

The data is arranged in ascending order

3, 4, 5, x, 8, 9, 11.

$\therefore$ x = 6.

iv) Given data is

15, 18, 10, 20, 23, 28, 12, 16

N = 8.

The data is arranged in ascending order

10, 12, 15, 16, 18, 20, 23, 28

First quartile, $Q^1$ = size of $\left(\frac{N+1}{4}\right)^{th}$ item

= size of $\left(\frac{8+1}{4}\right)^{th}$ item

= size of $(2.25)^{th}$ item

= $2^{nd}$ item + 0.25 ($3^{rd}$ item – $2^{nd}$ item)

= 12 + 0.25 (15 – 12)

= 12 + (0.25) (3) = 12.75.

v) two

vi) – 1

vii) Given

Coefficient of variance of x and y = 40
Cov (x, y) = 40
$\sigma_x^2 = 16$, $\sigma_y^2 = 256$

We know that

$$\text{Correlation coefficient} = \frac{\text{Cov}(x, y)}{\sigma_x \times \sigma_y}$$

$$r = \frac{40}{\sqrt{16.256}} = \frac{\cancel{40}^{10}5}{\cancel{4.16}_{8}} = \frac{5}{8}$$

viii) Given

$$P(A) : P(\bar{A}) = 3 : 7 = \frac{3}{10} : \frac{7}{10}$$

$$P(\bar{A}) : P(A) = 3 : 5 = \frac{3}{8} : \frac{5}{8}$$

$\therefore$ The probability that both pass = $\frac{3}{10} \times \frac{5}{8} = \frac{3}{16}$

ix) Given

N = 4, $\sigma = 2$
P = ½, q = 1 – p = 1 - ½ = ½
P (x = r) $= {}^nC_r \times q^{n-r} \times p^r$

$$= 6 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^2$$

$$= \frac{6}{16} = \frac{3}{8}$$

Probability that there are two heads = $\frac{3}{8}$

x) Given

np = 20
$\sqrt{npq} = 4$ => npq = 16

$$q = \frac{\cancel{np}q}{\cancel{np}} = \frac{16}{20}$$

$$q = \frac{4}{5}$$

xi) Given

n = 6, r = 3
p = ½, q = ½

$$p(x = 3) = {}^6C_3 \times \left(\frac{1}{2}\right)^{6-3} \times \left(\frac{1}{2}\right)^3$$

$$= 20 \times \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^3$$

$$= \frac{20}{64} = \frac{5}{16}$$

xii)

| Commodity | $p_0$ | $q_0$ | $p_1$ | $q_1$ | $p_0 q_0$ | $p_1 q_1$ | $p_0 q_1$ | $p_1 q_0$ |
|---|---|---|---|---|---|---|---|---|
| A | 4 | 3 | 6 | 2 | 12 | 12 | 8 | 18 |
| B | 5 | 4 | 6 | 4 | 20 | 24 | 20 | 24 |
| C | 7 | 2 | 9 | 2 | 14 | 18 | 14 | 18 |
| D | 2 | 3 | 1 | 5 | 6 | 5 | 10 | 3 |
| | | | | | 52 | 59 | 52 | 53 |

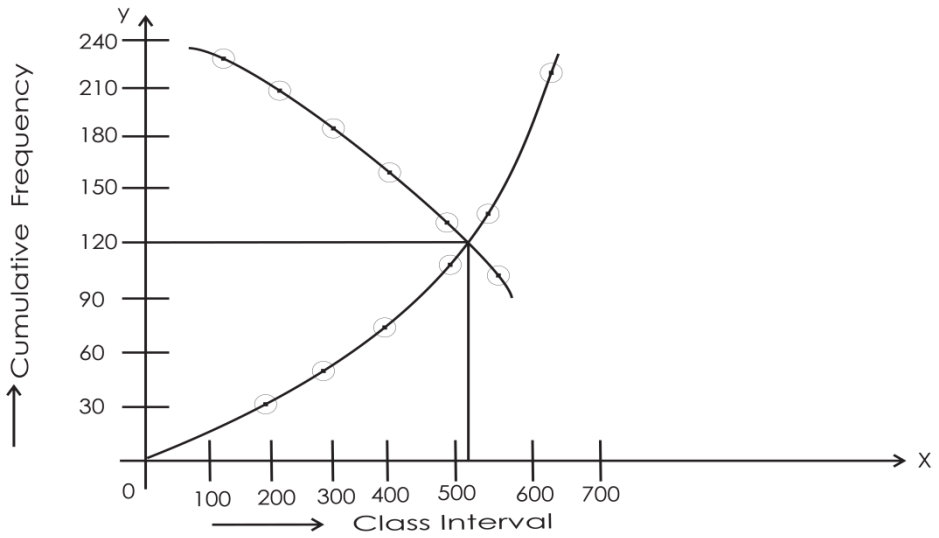
$$\text{Fisher's price index Number} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}} \times 100$$

$$= \sqrt{\frac{53}{52} \times \frac{59}{52}} \times 100 = 107.36$$

**Answer any FOUR of the following question** **[4 × 6 = 24]**

**3.**

| Class Interval (C.I.) | Frequency | Less than cumulative frequency | Greater than cumulative frequency |
|---|---|---|---|
| 100 – 200 | 12 | 12 | 240 |
| 200 – 300 | 18 | 30 | 228 |
| 300 – 400 | 30 | 60 | 210 |
| 400 – 500 | 42 | 102 | 180 |
| 500 – 600 | 60 | 162 | 138 |
| 600 - 700 | 78 | 240 | 78 |
| | 240 | | |




Median (M) $= L_1 + \frac{N_1 - c \times f}{f} \times c$

$= 500 + \frac{120 - 102}{60} \times 100$ $= 500 + 30$ $= 530.$

4.

| Class Interval (C.I.) | Frequency | Cumulative Frequency (C.F.) |
|---|---|---|
| 0 – 4 | 4 | 4 |
| 4 – 8 | 9 | 13 |
| 8 – 12 | 23 | 36 |
| 12 – 16 | 55 | 91 |
| 16 – 20 | 62 | 153 |
| 20 – 24 | 30 | 183 |
| 24 – 28 | 12 | 193 |
| 28 - 32 | 5 | 200 |
| | 200 | |

**5.**

| Marks in Economics | Marks in Accountancy | Rank in Economics ($R_E$) | Rank in Accountancy ($R_A$) | $D = R_E - R_A$ | $D^2$ |
|---|---|---|---|---|---|
| 48 | 62 | 6 | 4 | 2 | 4 |
| 60 | 78 | 3 | 1 | 2 | 4 |
| 72 | 65 | 1 | 3 | -2 | 4 |
| 62 | 70 | 2 | 2 | 0 | 0 |
| 56 | 38 | 4 | 7 | -3 | 9 |
| 40 | 54 | 7 | 6 | 1 | 1 |
| 39 | 60 | 8 | 5 | 3 | 9 |
| 52 | 32 | 5 | 8 | -3 | 9 |
| 30 | 31 | 9 | 9 | 0 | 0 |
| | | | | | $\Sigma D^2 = 40$ |

Given N = 9

Rank correlation, $r_k = 1 - \frac{6\sum D^2}{N^3 - N}$

$= 1 - \frac{6 \times 40}{9^3 - 9} = 1 - \frac{240}{720} = 1 - \frac{1}{3} = \frac{2}{3} = 0.67.$

$N_1 = \frac{N}{4} = \frac{200}{4} = 50$

$Q_1 = L_1 + \frac{N_1 - C \times f}{f} \times C$

$= 12 + \frac{50 - 36}{55} \times 4 = 12 + 1.018 = 13.018$

$N_1 = \frac{3N}{4} = \frac{3(200)}{4} = \frac{600}{4} = 150$

$Q_1 = L_1 + \frac{N_1 - C \times f}{f} \times C$

$= 16 + \frac{150 - 91}{62} \times 4 = 16 + 3.806 = 19.806$

Semi-inter quartile range ($S_1QR$) $= \frac{Q_3 - Q_1}{2}$

$= \frac{19.806 - 13.018}{2} = \frac{6.788}{2} = 3.394$

Coefficient of quartile deviation $= \frac{Q_3 - Q_1}{Q_3 + Q_1}$

$= \frac{19.806 - 13.018}{19.806 + 13.018} = \frac{6.788}{32.824} = 0.2068.$

**6.**

| Commodity | 2001 | | | 2005 | | | $p_0q_1$ | $p_1q_0$ |
|---|---|---|---|---|---|---|---|---|
| | $q_0$ | v | $p_0$ | $q_1$ | v | $p_1$ | | |
| A | 5 | 40 | 8 | 6 | 60 | 10 | 48 | 50 |
| B | 5 | 30 | 6 | 5 | 40 | 8 | 30 | 40 |
| C | 6 | 24 | 4 | 6 | 30 | 5 | 24 | 30 |
| D | 5 | 10 | 2 | 10 | 40 | 4 | 20 | 20 |
| | | 104 | | | 170 | | 122 | 140 |

Fisher's price index number,

$$p_{01} = \sqrt{\frac{\sum p_1q_0}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_0q_1}} \times 100 = \sqrt{\frac{140}{104} \times \frac{170}{122}} \times 100 = 136.96.$$

Time Reversal Test:

$$P_{01} \times P_{10} = \sqrt{\frac{\sum p_1q_0}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_0q_1} \times \frac{\sum p_0q_1}{\sum p_1q_1} \times \frac{\sum p_0q_0}{\sum p_1q_0}} = \sqrt{\frac{\cancel{140}}{\cancel{104}} \times \frac{\cancel{170}}{\cancel{122}} \times \frac{\cancel{122}}{\cancel{170}} \times \frac{\cancel{104}}{\cancel{140}}} = \sqrt{1} = 1.$$

$\therefore$ Time Reversal Test is satisfied.

Factors Reversal Test:

$$P_{01} \times Q_{10} = \sqrt{\frac{\sum p_1q_0}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_0q_1} \times \frac{\sum q_1p_0}{\sum q_0p_0} \times \frac{\sum q_1p_1}{\sum q_0p_1}}$$

$$= \sqrt{\frac{\cancel{140}}{104} \times \frac{170}{\cancel{122}} \times \frac{\cancel{122}}{104} \times \frac{170}{\cancel{140}}} = \sqrt{\left(\frac{170}{104}\right)^2} = \frac{170}{104} = \frac{\sum p_1q_1}{\sum p_0q_0}$$

$\therefore$ Factor Reversal Test is satisfied.

**7.**

| Year | Sales (y) | x | $x^2$ | xy | Trend values $y_c = a + bx$ |
|---|---|---|---|---|---|
| 2007 | 33 | -3 | 9 | -99 | 32.893 |
| 2008 | 35 | -2 | 4 | -70 | 42.643 |
| 2009 | 60 | -1 | 1 | -60 | 52.393 |
| 2010 | 67 | 0 | 0 | 0 | 62.143 |
| 2011 | 68 | 1 | 9 | 68 | 71.893 |
| 2012 | 82 | 2 | 4 | 164 | 81.643 |
| 2013 | 90 | 3 | 1 | 270 | 91.393 |
| | 435 | | 28 | 273 | |

Given N = 7

$$a = \frac{\sum y}{N} = \frac{435}{7} = 62.143$$

$$b = \frac{\sum xy}{\sum x^2} = \frac{273}{28} = 9.75$$

Estimate the sales for 2018 is

$y_c = 62.143 + 9.75(8) = 140.143.$

8. The probability of getting head, P = ½.
The probability of getting rail, q = 1 - ½ = ½.

Given P (X = 4), P (X = 5), P (X = 6) are in A.P.

2 P (X = 5) = P (X = 2), + P (X = 6)

$$2 \times {}^nC_5 \times (q)^{n-5} \times p^5 = {}^nC_4 \times q^{n-4} \times p^4 + {}^nC_6 \times q^{n-6} \times p^6$$

$$2 \times {}^nC_5 \times \left(\frac{1}{2}\right)^{n-5} \times \left(\frac{1}{2}\right)^5 = {}^nC_4 \times \left(\frac{1}{2}\right)^{n-4} \times \left(\frac{1}{2}\right)^4 + {}^nC_6 \times \left(\frac{1}{2}\right)^{n-6} \times \left(\frac{1}{2}\right)^6$$

$$2 \times {}^nC_5 \times \left(\frac{1}{\cancel{2}}\right)^{\cancel{n}} = \left(\frac{1}{\cancel{2}}\right)^{\cancel{n}} + \left({}^nC_4 + {}^nC_6\right)$$

$$2 \times {}^nC_5 = {}^nC_4 + {}^nC_6$$

$$2 \times \frac{n!}{(n-5)! \times 5!} = \frac{n!}{(n-4)! \times 4!} + \frac{n!}{(n-6)! \times 6!}$$

$$\frac{2}{(n-5)(n-6)!\times 5\times 4!} = \frac{1}{(n-4)(n-5)(n-6)!\times 4!} + \frac{1}{(n-6)!\times 6\times 5\times 4!}$$

$$\frac{2}{5(n-5)} = \frac{1}{(n-4)(n-5)} + \frac{1}{30}$$

$$\frac{2}{\cancel{5}(n-\cancel{5})} = \frac{30+(n-4)(n-5)}{\overset{6}{\cancel{30}}(n-4)(n-\cancel{5})}$$

$12(n-4) = 30 + (n-4)(n-5)$

$12n - 48 = 30 + n^2 - 9n + 20$

$N^2 - 21n\ 98 = 0$

$N^2 - 7n - 14n + 98 = 0$

$N(n-7) - 14(n-7) = 0$

$(n-7)(n-14) = 0$

$n - 7 = 0 \qquad n - 14 = 0$

$n = 7 \qquad n = 14$

$\therefore n = 7$ (or) 14.