Paper- 4: FUNDAMENTALS OF BUSINESS MATHEMATICS AND STATISTICS

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Full Marks: 100 **Time Allowed: 3 Hours**

Section – A

I. Answer any TWO questions. Each question carries 5 marks [2×5 = 10]

1. ∵ Sum P = ₹1,000, t = 5 years r = 12%

$\therefore$ Simple Interest (S. I) = $\frac{Prt}{100}$

$= \frac{1,000(12)}{100} \times 5$

$= 120 \times 5$

$= ₹600$

Now compound interest (C. I.) = $P\left\{(1+i)^n - 1\right\}$

$= 1,000\left\{\left(\frac{1+10}{100}\right)^5 - 1\right\}$

$= 1,000\left\{(1.1)^5 - 1\right\}$

$= 1,000\ (1.61051 - 1)$

$= 1,000\ (0.61051)$

$= ₹610.50$

Difference = C. I. – S. I. = 610.50 – 600

$= ₹10.50$

$\therefore$ He GOT ₹10.50 more as interest.

2. Given equation is $\sqrt{\frac{x}{x-1}} + \sqrt{\frac{1-x}{x}} = \frac{13}{6} \longrightarrow (i)$

Let $t = \sqrt{\frac{x}{x-1}}$ then $\sqrt{\frac{1-x}{x}} = \frac{1}{t}$

$\therefore$ From, (i), $t + \frac{1}{t} = \frac{13}{6}$

$\Rightarrow \frac{t^2+1}{t} = \frac{13}{6}$

$\Rightarrow 6t^2 + 6 = 13t$

$\Rightarrow 6t^2 - 13t + 6 = 0$

$\Rightarrow 6t^2 - 9t - 4t + 6 = 0$

$\Rightarrow 3t(2t - 3) - 2(2t - 3) = 0$

$\Rightarrow (2t - 3)(3t - 2) = 0$

$\therefore 2t - 3 = 0$ (OR) $3t - 2 = 0$

t = 3/2 (OR) t = 2/3

Case (i) when t = 3/2

$$\Rightarrow \sqrt{\frac{x}{x-1}} = \frac{3}{2}$$

S. O. B

$$\frac{x}{x-1} = \frac{9}{4}$$

$\Rightarrow 4x = 9x - 9$

$\Rightarrow 5x = 9$

$X = 9/5$

Case (ii) when t = 2/3

$$\Rightarrow \sqrt{\frac{x}{x-1}} = \frac{2}{3}$$

S. O. B

$$\frac{x}{x-1} = \frac{4}{9}$$

$\Rightarrow 9x = 4x - 4$

$\Rightarrow 5x = -4$

$X = -4/5$

3. Given demand function faced by firm is P = 500 – 0.2x and cost function is c = 25x + 10,000

Now, total Revenue (R) = Px

$= (500 - 0.2x)\ x$

$= 500x - 0.2x^2$

$\therefore$ Profit (P) = R – C

$= 500x - 0.2x^2 - 25x - 10{,}000$

$= -0.2x^2 + 475x - 10{,}000$

For Maximum profit, MR = MC

$$\Rightarrow \frac{dR}{dx} = \frac{dc}{dx}$$

$\Rightarrow 500 - (0.2)\ (2x) = 25$

$\Rightarrow 500 - (0.4)x = 25$

$$\Rightarrow (0.4)x = \frac{500}{25} = 20$$

$$x = \frac{20}{0.4} = 50 \text{ units}$$

$\therefore$ Price at x = 50 units is

P = 500 – (0.2) (50)

= 500 – 10

= ₹490

II. Answer any TWO questions. Each question carries 3marks **[2 × 3 = 6]**

4. Given Condition is

$$\frac{\cancel{(x-1)}\ (x+1)}{(x+2)\ \cancel{(x-1)}} = \frac{9}{10}$$

$$\Rightarrow \quad \frac{x+1}{x+2} = \frac{9}{10}$$

$$\Rightarrow \quad 10x + 10 = 9x + 18$$

$$\Rightarrow \quad X = 8$$

5. Given n (A) = 41, n (B) = 19 and n(A ∩ B) = 10

We know that n (A ∪ B) = n(A) + n (B) – n (A ∩ B)

⇨ n (A ∪ B) = 41 + 19 – 10

= 60 – 10

n (A ∪ B) = 50

6. $\lim_{x \to o} \frac{a^x - b^x}{x}$

$$= \lim_{x \to o} \frac{a^x - b^x + 1 - 1}{x}$$

$$= \lim_{x \to o} \frac{(a^x - 1) - (b^x - 1)}{x}$$

$$= \lim_{x \to o} \frac{a^x - 1}{x} - \lim_{x \to o} \frac{b^x - 1}{x}$$

= log a – log b

$= \log_e \left(\frac{a}{b}\right)$

III. Choose the correct answer **[5 × 1 = 5]**

7. Answer: (c)

Let $\frac{A}{3} = \frac{B}{4} = \frac{c}{5} = k$(say)

∴ A = 3k, B = 4k, C = 5k

Now A: B: C = 3K : 4K : 5K

= 3: 4: 5

8. Answer: (b)

Required no. of ways = $4^5 = 1024$

9. Answer: (a)

$\log_2 (\log_5 625)$

$= \log_2 (\log_5 5^4)$

$= \log_2 4 . \log_5 5)$ $\quad \because \log_5 5 = 1$

$= \log_2 4$

$= \log_2 2^2$

$= 2 \log_2 2 = 2(1) = 2$

10. Answer: (a)

$\because f(x-1) = 2x-3$

By verification west

Take f (x) = 2x – 1

Now f (x – 1) = 2(x – 1) – 1

= 2x – 3

11. Answer: (b)

$$\int (x+3)^6 dx = \frac{(x+3)^{6+1}}{6+1} + C$$

$$= \frac{(x+3)^7}{7} + C$$

IV. Fill in the blanks **[5 ×1= 5]**

12. Answer: 27 : 8

$\because b \alpha a^3$

$\Rightarrow b = k\, a^3$

$= k\left(\frac{27}{8}\right)$

$\therefore$ b increases in the ratio 27: 8

13. Answer: ₹750

r = 6% p.a. P = ₹2,500, t = 5 years

$$S.I = \frac{Prt}{100}$$

$$= \frac{2,5\cancel{00} \times 6 \times 5}{\cancel{100}}$$

= 125 × 6

= ₹750

14. Answer: 5

$\therefore$ x = 8 y = 27

$$\text{Now} \left(x^{4/3} + y^{2/3}\right)^{1/2}$$

$$= \left(8^{4/3} + 27^{2/3}\right)^{1/2}$$

$$= \left[(2^3)^{4/3} + (3^3)^{2/3}\right]^{1/2}$$

$$= \left(2^4 + 3^2\right)^{1/2}$$

$= (16 + 9)^{½}$

$= (25)^{½}$

$= (5^2)^{½} = 5$

15. Answer: $3^x \log 3$

$\because y = 3^x$

Now $\frac{dy}{dx} = 3^x \log 3$ $\quad (\because \frac{d}{dx}(a^x) = a^x \log a)$

16. Answer: $x^3 + c$

$$\int 3x^2 dx = \cancel{3}\left(\frac{x^3}{\cancel{3}}\right) + c$$

$$= x^3 + c$$

V. State whether the following statements are true or false **[5×1= 5]**

17. False
18. False
19. True
20. False
21. True

VI. Match the following **[5× 1 = 5]**

22. Answer: (C)
The sub – duplicate ratio of 49 : 81
= √49 : √ 81
= 7 : 9

23. Answer: (E)
AB = $[5 \ \ 2] \begin{bmatrix} 3 \\ 4 \end{bmatrix}$
= [23]

24. Answer (A)
$\because x = 2$ and $3x - 2y = 5$
⇨ 3(2) – 2y = 5
⇨ 2y = 6 – 5 = 1
⇨ Y = ½

25. Answer: (B)
$n_{cr} + n_{cr-1} = n+1_{cr}$

26. Answer: (D)
$\lim_{x \to 2} 3x + 6$
= 3(2) + 6 = 6 + 6 = 12

VII. Answer the following in one or two steps **[9×2] = 18**

27. Truth table for " p∧q "

P	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

28. Given is equation is
4x + 4 < 2x +3
⇨ 2x < - 1

$\Rightarrow x < -\frac{1}{2}$

29. $\because {}^{11}C_x = {}^{11}C_y$ $\quad \left[\because {}^{n}C_r = {}^{n}C_s \Rightarrow n = r+s\right]$

$\Rightarrow 11 = x + y$

30. $\int_1^4 6\,dx = 6\int_1^4 dx$

$= 6[x]_1^4$

$= 6\,(4 - 1)$

$= 6 \times 3 = 18$

Section – B

1. Answer any Nine questions of the following [9× 2 = 18]
Each question carries 2 marks

(i) Answer: (d)

The upper part of a table that describes the column and sub-column

(ii) Answer: (a)

Grouped frequency distribution

(iii) Answer: (c)

Given

Correct items = 43,32
Incorrect items = 34,23
N = 50
Average, $\bar{x} = 38$
Correct total = 50 × 38 + 43 + 32 – 34 – 23 = 1948

Correct mean = $\frac{1948}{50} = 38.36$

(iv) Answer: (d)

Given the equation is 5x + 6y = 70
Median of x = 8
6 Median of y = 70 – 5, median of x
Median of y = $\frac{1}{6}$. (70 - 40)

$= \frac{30}{6}$

Median of y = 5.

(v) Answer: (b)

Given

N = 100 $\bar{x}$ = 50 , M = 40, Z = ?

Mode (z) = 3M - 2$\bar{x}$

= 3(40) – 2(50)

= 120 – 100 = 20

(vi) Answer: (a)

Given

$r_k = 0.6$, $\sum D^2 = 66$, N = ?

$$r_k = 1 - \frac{6\sum D^2}{N^3 - N}$$

$$0.6 = 1 - \frac{6(66)}{N^3 - N}$$

$$\frac{396}{N^3 - N} = 1 - 0.6 = 0.4$$

$$N^3 - N = \frac{396}{0.4} = 990$$

$N^3 - N = 990$

$N(N^2 - 1) = 990$

$\therefore N = 10$

(vii) Answer: (b)

The regression lines are

$X + 3y = 7$ ---------(1)

$2x + 5y = 12$ -------(2)

The point of intersection of (1) and (2) is (1, 2)

(viii) Answer: (b)

We know that the total probability is one

P (A) = 1/3, P (B) = 0, P (C) = 2/3

(ix) Answer: (b)

Given P (A) = $\frac{1}{2}$, P (B) = $\frac{3}{5}$

A and B are independent events. Then

$P(A \cap B) = P(A), P(B) = \frac{1}{2} \cdot \frac{3}{5} = \frac{3}{10}$

(x) Answer: (b)

Given np = 9

Npq = 2.25

$q = \frac{npq}{np} = \frac{2.25}{9} = \frac{1}{4}$

$p = 1 - q = 1 - \frac{1}{4} = \frac{3}{4}$

$np = 9$

$n\left(\frac{3}{4}\right) = 9 \quad 3 \quad \Rightarrow \quad n = 12$

(xi) Answer: (a)

Given $n = 5$; $r = 3$

$P = \frac{1}{2}$; $q = 1 - p = \frac{1}{2}$

$P(x = r) = {}^{n}c_{r} \cdot q^{n-r} \cdot p^{r}$

$P(x = 3) = {}^{5}c_{3} \cdot \left(\frac{1}{2}\right)^{5-3} \cdot \left(\frac{1}{2}\right)^{3}$

$= 10 \cdot \frac{1}{4} \cdot \frac{1}{8}$

$= \frac{10}{32}$

$= \frac{5}{16} = 0.3125$

(xii) Answer: (a)

Commodity	P_0	Q_0	P_1	Q_1	P_1q_1	P_0q_0
A	4	3	6	2	12	12
B	5	4	6	4	24	20
C	7	2	9	2	18	14
D	2	3	1	5	5	6
					59	52

Factor reversal test : -

$P_{01} \times Q_{01} = \frac{\Sigma P_1q_1}{\Sigma P_0q_0}$

$= \frac{59}{52}$

2. Answer any Nine question of the following [9×2 = 18]
Each question carries 2 marks

i) Histogram

ii)

Class Interval (C. I)	Cumulative Frequency (C. F)	Frequency (F)
200 – 250	56	18
250 – 300	38	23

300 – 350	15	15
350 - 400	0	0

∴ The number of observations between 250 and 300 is 23.

iii) Given date

1, 3, 5, 6, x, 10

Arithmetic mean = 6

$= \frac{1+3+5+6+x+10}{6} = 6$

$= x + 25 = 36$

$\therefore x = 36 - 25$

$x = 11$

iv)

x	f	x. f
1	1	1
2	2	4
3	3	9
4	4	16
5	5	25
	15	55

Arithmetic Mean = $\frac{\sum f \cdot x}{\sum f} = \frac{55}{15} = \frac{11}{3} = 3.67$

v) Given

Arithmetic Mean (A. M) = 6.5

Geometric Mean (G. M) = 6

We know that

$(G. M.)^2 = (A. M.) \times (H. M.)$

$G^2 = AH$

$36 = 6.5 \times (H. M.)$

Harmonic Mean = $\frac{36}{6.5} = 5.54$

vi) Regression equation are

$8x - 10y + 66 = 0$

$10y = 8x + 66$

$y = \frac{8}{10}x + 6.6$

$\therefore b_{yx} = \frac{8}{10}$

$40x - 18y = 214$

$40x = 18y + 214$

$x = \frac{18}{40}y + \frac{214}{40}$

$\therefore b_{xy} = \frac{18}{40}$

We know that

Coefficient of wrrolation, $r = \pm\sqrt{b_{xy}.b_{yx}}$

$$= \pm \sqrt{\frac{\cancel{18}\ 9}{\cancel{40}} \cdot \frac{\cancel{8}\ 1}{\cancel{10}}}$$

$$= \frac{3}{5}$$

vii) Zero

viii) Given

$P(A \cap B) = 0.60 \quad P(A \cup B) = 0.70$

We know that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A) + P(B) = 0.6 + 0.7 = 1.3$

$$1 - P\left(\bar{A}\right) + 1 - P\left(\bar{B}\right) = 1.3$$

$$P\left(\bar{A}\right) + P\left(\bar{B}\right) = 2 - 1.3$$

$$P\left(\bar{A}\right) + P\left(\bar{B}\right) = 0.7$$

ix) $P(A) = 1/2, \quad P(B) = 1/3, \quad P(AB) = 1/4$

We know that

$$P\left(\frac{A}{B}\right) = \frac{P(AB)}{P(B)}$$

$$= \frac{\frac{1}{4}}{\frac{1}{3}} = \frac{3}{4}$$

$$P\left(\frac{A}{B}\right) = \frac{3}{4}$$

x) We know that

$$P(x = r) = \frac{e^{-m} \cdot m^r}{r!}$$

Given

$P(x = 1) = P(x = 2)$

$$\frac{\cancel{e^{-m}} \cdot \cancel{m^r}}{1!} = \frac{\cancel{e^{-m}} \cdot m^r}{2!}$$

$M = 2$

$\therefore$ Mean = 2

xi) $n(s) = 5$

$\therefore$ The probability of getting the value 8 is $P(x = 8) = 1/5$

xii)

Group	Group Index (I)	Weight (w)	WI
A	120	6	720
B	132	3	396
C	98	4	392

D	115	2	230
E	108	1	108
F	95	4	380
		20	2226

$$\text{General index} = \frac{\sum W.I}{EW} = \frac{2226}{20} = 111.3$$

Answer any FOUR of the following questions **(4 × 6 = 24)**

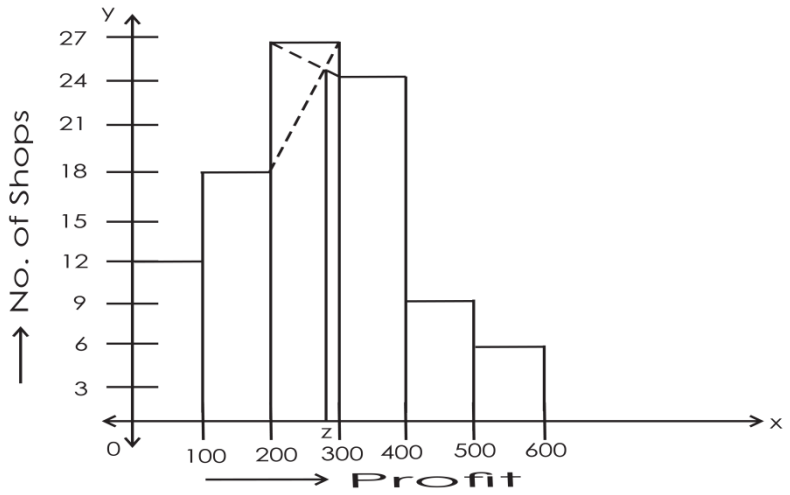
3.

Profit (₹)	No. of Shops
0 – 100	12
100 – 200	18
200 – 300	27
300 – 400	24
400 – 500	10
500 - 600	6

$L_1 = 200$; $f_1 = 27$;
$f_0 = 18$; $f_2 = 24$; $c = 100$

$$\text{Model (Z)} = L_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times c$$

$$= 200 + \frac{27 - 18}{54 - 18 - 24} \times 100 \qquad = 200 + \frac{9}{12} \times 100 \qquad = 275.$$



∴ From Graph Modal Value (z) = 275

4.

Class Interval (C.I.)	Frequency (f)	Mid value (m)	f.m.	Cumulative Frequency (C.F.)
25 – 50	21	37.5	787.5	21
50 – 75	47	62.5	2937.5	68
75 – 100	67	87.5	5862.5	135
100 – 125	89	112.5	10012.5	224
125 – 150	55	137.5	7562.5	279

150 – 175	21	162.5	3412.5	300
	300		30575	

Mean $\bar{x} = \frac{\sum f \times m}{\sum f} = \frac{30575}{300} = 101.9166$

Median (M) = $N_1 = \frac{N}{2} = \frac{300}{2} = 150$

$= L_1 + \frac{N_1 - C \times f}{f} \times C \quad = 100 + \frac{150 - 135}{89} \times 25 \quad = 101.2135.$

5.

Class Interval (C.I.)	Frequency (f)	Mid value (m)	f.m.	$dx = m - \bar{x}$	dx^2	fdx^2
0 – 10	5	5	25	-30.25	915.0625	4575.3125
10 – 20	15	15	225	-20.25	410.0625	6150.9375
20 – 30	30	25	750	-10.25	105.0625	3151.875
30 – 40	65	35	2275	-0.25	0.0625	4.0625
40 - 50	80	45	3600	9.75	95.0625	7605
	195		6875			21487.1875

Mean $\bar{x} = \frac{\sum f \times m}{\sum f} = \frac{6875}{195} = 35.25$

Standard Deviation (σ) = $\sqrt{\frac{\sum f \times dx^2}{\sum f}} \times 100 \quad = \sqrt{\frac{21487.1875}{195}} \quad = 10.497.$

Coefficient of Variation = $\frac{\text{Standard Deviation}}{\text{Mean}} \times 100 = \frac{\sigma}{\bar{x}} \times 100 = \frac{10.497}{35.25} \times 100 = 29.78.$

6.

x	y	$x = x - \bar{x}$	$y = y - \bar{y}$	xy	x^2	y^2
2	7	-1.875	4	-7.5	3.5156	16
6	2	2.125	-1	-2.125	4.5156	1
4	1	0.125	-2	-0.25	0.0156	4
3	1	-0.875	-2	1.75	0.7656	4
2	2	-1.875	-1	1.875	3.5156	1
2	3	-1.875	0	0	3.5156	0
8	2	4.125	-1	-4.125	17.0156	1
4	6	0.125	3	0.375	0.0156	9
				-10	32.8748	36

$\bar{x} = \frac{\sum x}{N} = \frac{31}{8} = 3.875$

$\bar{y} = \frac{\sum y}{N} = \frac{24}{8} = 3$

By x = $\frac{\sum xy}{\sum x^2} = \frac{-10}{32.8748}$

Regression equation of y on x is

$y - \bar{y} = Byx\ (x - \bar{x})$

$y - 3 = \frac{-10}{32.8748}\ (x - 3.875)$

$y = 3 - \frac{10}{32.8748}\ (x - 3.875)$

If x = 20, then

$y = 3 - \frac{10}{32.8748}(20 - 3.875)$

$= 3 - 4.9$

$= -1.9.$

7.

Commodity	2001		2005		p_0q_0	p_1q_1	p_0q_1	p_1q_0
	p_0	q_0	p_1	q_1				
A	5	10	4	12	50	48	60	40
B	8	6	7	7	48	49	56	42
C	6	3	5	4	18	20	24	15
					116	117	140	97

Fisher's Price Index Number,

$$p_{01} = \sqrt{\frac{\sum p_1q_0}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_0q_1}} \times 100 = \sqrt{\frac{97}{116} \times \frac{117}{140}} \times 100 = 83.59$$

Marshall Edgeworth price Index Number,

$$p_{01} = \frac{\sum p_1q_0 + \sum p_1q_1}{\sum p_0q_0 + \sum p_0q_1} \times 100 = \frac{97+117}{116+140} \times 100 = \frac{214}{256} \times 100 = 83.59$$

8. Given

The probability of A will succeed, $P(A) = \frac{4}{5}$

The probability of B will succeed, $P(B) = \frac{3}{4}$

The probability of C will succeed, $P(C) = \frac{2}{3}$

Now,

$P(A) = \frac{4}{5} \Rightarrow P(\bar{A}) = 1 - P(A) = 1 - \frac{4}{5} = \frac{1}{5}$

$P(B) = \frac{3}{4} \Rightarrow P(\bar{B}) = 1 - P(B) = 1 - \frac{3}{4} = \frac{1}{4}$

$P(C) = \frac{2}{3} \Rightarrow P(\bar{C}) = 1 - P(C) = 1 - \frac{2}{3} = \frac{1}{3}$

$\therefore$ The probability that atleast two of them hit the balloon

$= P(A \cap B \cap \bar{C}) + P(A \cap \bar{B} \cap C) + P(\bar{A} \cap B \cap C) + P(A \cap B \cap C)$

$= P(A) \times P(B) P(\bar{C}) + P(A) \times P(\bar{B}) \times P(C) + P(\bar{A}) \times P(B) \times P(C) + P(A) \times P(B) \times P(C)$

$= \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3} + \frac{4}{5} \times \frac{2}{3} \times \frac{1}{5} \times \frac{3}{4} \times \frac{2}{3} + \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3}$

$= \frac{12}{60} + \frac{8}{60} + \frac{6}{60} + \frac{24}{60}$

$= \frac{50}{60}$

$= \frac{5}{6}$