FOUNDATON EXAMINATION

(REVISED SYLLABUS - 2008)

Paper - 4 : BUSINESS MATHEMATICS & STATISTICS FUNDAMENTALS

Section - I

[Arithmetic]

Q. 1. (a) If $\frac{x}{b+c}=\frac{y}{c+a}=\frac{z}{a+b}$ then show that

$(b-c)(x-a)+(c-a)(y-b)+(a-b)(z-c)=0$

(b) Two numbers are in the ratio of 3:4. If 10 is subtracted from both of them then the ratio becomes 1:3. The numbers are :

(i) 9 and 12 (ii) 12 and 16 (iii) 15 and 20 (iv) none of these

Answer 1. (a)

Let $\frac{x}{b+c}=\frac{y}{c+a}=\frac{z}{a+b}$ = k (constant). say

Then x = k(h + c), y= k(c + a), z = k(a + b)

So, $(b-c)(x-a)+(c-a)(y-b)+(a-b)(z-c)$

$= [x(b-c)+y(c-a)+z(a-b)] - [a(b-c)+b(c-a)+c(a-b)]$

$= [k(b+c)(b-c)+k(c+a)(c-a)+k(a+b)(a-b)] - [ab-ac+bc-ab+ac-bc]$

$= [k(b^2-c^2)+k(c^2-a^2)+k(a^2-b^2)] - 0$

$= [k(b^2-c^2+c^2-a^2+a^2-b^2)] - 0$

$= k \times 0 - 0 = 0 - 0 = 0$ Proved

Answer 1. (b)

Let the numbers be 3k and 4k

Now $\frac{3k-10}{4k-10}=\frac{1}{3} \Rightarrow 9k-30=4k-10$

$\Rightarrow 5k=20 \Rightarrow k=4$

So the numbers are 3 × 4 =12, and 4 × 4 = 16

Q. 2. (a) The average marks in "Elements of Mathematics" of Preliminary students of 3 centres in India is 50. The number of candidates in 3 centres are respectively 100, 120, 150. If the averages of the first two centres are 70 and 40, find the average marks of the third centre.

(b) A class has 3 divisions. Average marks of the students of the class, first division, second division and third division are 47, 44, 50 and 45 respectively in Mathematics. If first two division have 30 and 40 students, find the number of students in third division when all the students of the class have Mathematics as a subject.

(c) Mean monthly income of 10 workers in factory A is ₹ 4,000 and that of workers in factory B is ₹ 3,700. If the mean income of all workers in A and B is 3,800 per month, find the number of workers in B.

Answer 2. (a)

Let the average marks of the third centre be .

Then using the formula : $\bar{x} = \dfrac{n_1\bar{x} + n_2\bar{x}_2 + n_3\bar{x}_3}{n_1 + n_2 + n_3}$, we get

$$50 = \frac{100 \times 70 + 120 \times 40 + 150 \times \bar{x}_3}{100 + 120 + 150}, \quad \text{or, } 50 = \frac{11{,}800 + 150\bar{x}_3}{370},$$

or, $11{,}800 + 150\,\bar{x}_3 = 18{,}500$, or, $150\,\bar{x}_3 = 18{,}500 - 11{,}800 = 6{,}700$

$$\therefore \quad \bar{x}_3 = \frac{6{,}700}{150} = 44.67$$

Hence the required average marks of the third centre = 44.67.

Answer 2. (b)

Let the no. of students be x in the third division.

Students have total marks in Mathematics in 1st division = 30 × 44 = 1320

Students have total marks in Mathematics in 2nd division = 40 × 50 = 2000

Students have total marks in Mathematics in 3rd division = x × 45 = 45x

Total marks in Mathematics in the whole class = 3320 + 45x

Total number of students in the class = 30 + 40 + x = 70 + x

Then average marks of the students in the class =

Thus $\dfrac{3320 + 45x}{70 + x}$, = 47

So 3320 + 45x = 3290 + 47x

or, 30 = 2x

or, x = 15

Answer 2. (c)

Let n_2 be the no. of workers in B. Then $\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$.

Here $n_1 = 10$, $\bar{x}_1 = 4000$, $\bar{x}_2 = 3{,}700$ and $\bar{x} = 3{,}800$, $n_2 = ?$

$\therefore$ $3800 = \frac{10 \times 4000 + n_2 \times 3700}{10 + n_2}$, or, $38000 + 3800n_2 = 40000 + 3700n_2$,

or $100n_2 = 2000$, or, $n_2 = 20$.

Hence the required no. of workers in B = 20.

Q. 3. (a) A vessel contains a mixture of Wine and Water. Had there been a litre more of Wine and a litre less of Water, the ratio of Wine to Water would have been 7 : 8; but had there been a litre more of Water and a litre less of Wine, the ratio would have been 2 : 3. How many litres does the mixture consist of?

(b) The proportion of liquid I and II in four samples are 2:1, 3:2, 5:3 and 7:5. A mixture is prepared by taking equal quantities of the samples. Find the ratio of liquid I to liquid II in the final mixture.

Answer 3. (a)

Let the vessel contain *x* litres of milk and *y* litres of water. Then the vessel contains $(x + y)$ litres of mixture.

By the given conditions,

$$\frac{x+1}{y-1} = \frac{7}{8} \qquad \text{(i)}$$

and $$\frac{x-1}{y+1} = \frac{2}{3}. \qquad \text{(ii)}$$

$\therefore$ From (i),

$8x + 8 = 7y - 7$,

or, $8x - 7y = -15$. (iii)

From (ii),

$3x - 3 = 2y + 2$,

or, $3x - 2y = 5$. (iv)

Solving (iii) and (iv), we get $x = 13$ and $y = 17$.

Hence the vessel contains 13 + 17, i.e., 30 litres of mixture.

Answer 3. (b)

$$\frac{\text{Liquid I}}{\text{Liquid II}} = \left[\frac{2}{3}x + \frac{3}{5}x + \frac{5}{8}x + \frac{7}{12}x\right] \Big/ \left[\frac{1}{3}x + \frac{2}{5}x + \frac{3}{8}x + \frac{5}{12}x\right]$$

$$= \frac{80+72+75+70}{40+48+45+50} = \frac{297}{183} = \frac{99}{61}$$

$\therefore$ Ratio of Liquid I and Liquid II in final mixture is 99:61.

Q. 4. (a) If $\frac{\alpha}{q-r} = \frac{\beta}{r-p} = \frac{\gamma}{p-q}$ then prove that $\alpha+\beta+\gamma = 0 = p\alpha + q\beta + r\gamma$.

(b) A sum deposited at a bank fetches ₹ 13,440 after 5 years at the rate of 12% simple interest. Find the principal amount.

(c) If I ask you for a loan and agree to repay you ₹ 300 after nine months from today, how much should you loan me if you are willing to make the loan at the rate of 6% p.a.?

Answer 4. (a)

Let $\frac{\alpha}{q-r} = \frac{\beta}{r-p} = \frac{\gamma}{p-q} = k$,

or, $\alpha = k(q-r)$;

$\beta = k(r-p)$;

$\gamma = k(p-q)$

$\alpha+\beta+\gamma = k(q-r+r-p+p-q) = 0$;

$p\alpha + q\beta + r\gamma$

$pk(q-r) + qk(r-p) + rk(p-q)$

or, $k(pq - pr + qr - pq + pr - qr) = 0$

$\therefore \alpha+\beta+\gamma = 0 = p\alpha + q\beta + r\gamma$

Answer 4. (b)

Let the principal amount be ₹ 100. Then

Simple interest on ₹ 100 for 5 years at 12% p.a. = 12 × 5 = ₹ 60.

Amount at the end of 5 years = 100 + 60 = ₹ 160.

Amount	Principal
160	100
13,440	x

$\therefore$ x = 100 × $\frac{13,440}{160}$ = ₹ 8,400

Answer 4. (c)

If ₹ 100 be the amount of loan, then interest $= 6 \times \frac{9}{12} =$ ₹ $\frac{9}{2}$ and amount with interest $= 100 + \frac{9}{2} =$ ₹ $\frac{209}{2}$.

If repayable amount be ₹ $\frac{209}{2}$, then amount of loan is ₹ 100.

If repayable amount be ₹ 300, then amount of loan is ₹ $100 \times \frac{2}{209} \times 300 =$ ₹ 287.08.

Q. 5. (a) A bill was drawn on 14 June 1989 at 8 months after date and was discounted on 24 September 1984 at 5% p.a. If the banker's gain on the basis of simple interest is ₹ 3, for what sum the bill was drawn?

(b) If the difference between true discount and banker's discount on a sum due in 3 months 4% per annum is ₹ 20, find the amount of bill.

(c) The Bill Value (B.V.) of a bill is ₹ 1,01,000. Find the Banker's Gain (B.G.) after 73 days at 5% p.a.

Answer 5. (a)

Date of drawing	14.6.84
Period	8 months
Nominal due date	14.2.85
Days of grace	3 days
Legally due date	17.2.85

$\therefore$ Unexpired period = 24.9.84 to 17.2.85 = 6 + 31 + 30 + 31 + 31 + 17 = 146 days.

Given B. G. = ₹ 3, or, B.D. – T.D. = ₹ 3 ... (i)

If P. V. = ₹ 100, then T.D. = 5 × $\frac{146}{365}$ = ₹ 2, B. V. = P. V. + T. D. = 100 + 2 = ₹ 102.

$\therefore$ B. D. = Interest on B. V. = $\frac{5}{100}\times102\times\frac{146}{365}=\frac{204}{100}$ = ₹ 2.04.

$\therefore$ B. G. = B. D. – T. D. = ₹ 2.04 – ₹ 2 = ₹ .04 = ₹ $\frac{1}{25}$.

If B. G. = ₹ $\frac{1}{25}$, then B. V. = ₹ 102.

$\therefore$ If B. G. = ₹ 3, then B. V. = $\frac{102}{1/25}\times3$ = ₹ 7,650.

Hence the required **Bill Value** is ₹ **7,650**.

Answer 5. (b)

A = Amount due at the end of n years = P (1 + ni) where

P = Present value, i = rate of interest, $n=\frac{3}{12}=\frac{1}{4}$ year

BD = Ani = P (1 + ni) ni, TD = Pni

BD – TD = P(ni)² $=20\Rightarrow P=\frac{20}{\left(\frac{1}{4}\times\frac{4}{100}\right)^2}=20\times100^2$ = 200000 (in ₹)

A = P (1 + ni) = 200000 $\left(1+\frac{1}{4}\times\frac{4}{100}\right)$ = 200000 × 1.01 =202000

Amount of Bill : ₹ 202000.

Answer 5. (c)

$Pv=\frac{BV}{1+ni}=\frac{101000}{1+\frac{72}{265}\times\frac{5}{10c}}$ = 100000

BD =BV × ni = 101000 × 0.01 = ₹ 1010

TD= PV × ni = 100000 × 0.01 = ₹ 1000

BG = BD-TD = ₹ 10

Section - II

[Algebra]

Q. 6. Choose the correct option showing necessary reasons/calculations.

(a) **Solution of** $(\sqrt[3]{2})^{2x+7} = (\sqrt[4]{2})^{7x+2/3}$ **is**

(i) x = 1, (ii) x = 3, (iii) x = 4, (iv) none of these

(b) $^{n}C_{r} + ^{n}C_{r-1}$ **is equal to**

(i) $^{n-1}C_{r}$**, (ii)** $^{n+1}C_{r}$**, (iii)** $^{n}C_{r+1}$**, (iv) none of these.**

(c) If $\dfrac{\log x}{y-z} = \dfrac{\log y}{z-x} = \dfrac{\log z}{x-y}$ **then the value of xyz is**

(i) 1, (ii) 0, (iii) – 1, (iv) none of these.

Answer 6. (a)

(iv) $(2^{1/3})^{2x+7} = (2^{1/4})^{7x+2/3}$

$$\therefore \quad \frac{2x+7}{3} = \frac{7x+\frac{2}{3}}{4}$$

or, $8x + 28 = 21x + 2.$

or, $21x - 8x = 28 - 2.$

or, $13x = 26$

$x = 2.$

Answer 6. (b)

(ii)

$$^{n}C_{r} + ^{n}C_{r-1} = \frac{\underline{|n}}{\underline{|r}\,\underline{|n-r}} + \frac{\underline{|n}}{\underline{|r-1}\,\underline{|n-r+1}}$$

$$= \frac{\underline{|n}}{\underline{|r}\,\underline{|n-r}} + \left[\frac{n-r+1+r}{r(n-r+1)}\right]$$

$$= \frac{(n+1)\underline{|n}}{\underline{|r}\,\underline{|n-r+1}} = \frac{\underline{|n+1}}{\underline{|r}\,\underline{|n-r+1}} = {}^{n+1}C_{r}\,.$$

Answer 6. (c)

(iv)

$$\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y} = K \text{ (say)}$$

Then $\log x = K(y-z)$

$\log y = K(z-x)$

$\log z = K(x-y)$

Adding $\log x + \log y + \log z = K\,(y - z + z - x + x - y) = K \times 0 = 0$

or, $\log xyz = \log 1$

or, $xyz = 1$.

Q. 7. (a) If *w* be an imaginary cube root of unity find out the value of $(1-w)(1-w^2)(1-w^4)(1-w^8)$.

(b) Simple interest and compound interest in 2 years for same principal are Rs. 200 and Rs. 210 at the same rate of interest per annum. Find the principal amount.

(c) The volume of a gas varies directly as the absolute temperature and inversely as pressure. When the pressure is 15 units and the temperature is 260 units the volume is 200 units. What will be the volume when the pressure is 18 units and the temperature is 195 units?

Answer 7. (a)

$(1-w)(1-w^2)(1-w^4)(1-w^8)$

$= (1-w)(1-w^2)(1-w)(1-w^2)$

$= (1-w)^2(1-w^2)^2$

$= (1+w^2-2w)(1+w^4-2w^2) = (-3w)(-3w^2) = 9w^3 = 9$

Answer 7. (b)

Let x = Principal amount and r % = rate of interest per annum

The simple interest = Rs. 200 = $x \times \frac{r}{100} \times 2 = \frac{rx}{50} \Rightarrow rx = 10000$

The compound interest = Rs. 210 = $x\left(1+\frac{r}{100}\right)^2 - x$

$$= x\left(\frac{2r}{100}+\frac{r^2}{10000}\right) = \frac{rx}{50}+\frac{r^2x}{10000}$$

$$\Rightarrow 210 = 200 + \frac{r^2x}{10000} \Rightarrow 10 = \frac{10000r}{10000} \Rightarrow r = 10$$

So, $x = \frac{200 \times 50}{r} = \frac{10000}{10} = 1000$ Rs.

Answer 7. (c)

Volume = V, Pressure = P, Absolute Temp = T

$\therefore\ V \propto T \ \&\ V \propto \frac{1}{P} \Rightarrow V \propto \frac{T}{P} \Rightarrow V = K\frac{T}{P}$ K = constant

Then P = 15, T = 260 then V = 200

$200 = K\,\frac{260}{15} \Rightarrow K = \frac{150}{13}$

When P = 15, T = 260 then $V = \frac{150}{13} \times \frac{195}{18} = 125$ units

Q. 8. (a) If $p = \log_{10} 20$ and $q = \log_{10} 25$, find x and such that $2 \log_{10} (x + 1) = 2p - q$

(b) If $x = \log_{2a} a$, $y = \log_{3a} 2a$, $z = \log_{4a} 3a$, Show that : $xyz + 1 = 2yz$.

(c) Show that $\log_3 \sqrt{3\sqrt{3\sqrt{3.....\infty}}} = 1$.

Answer 8. (a)

$2p - q = 2 \log_{10} 20 - \log_{10} 25 = \log_{10} (20)^2 - \log_{10} 25$

$= \log_{10} 400 - \log_{10} 25 = \log_{10} \dfrac{400}{25} = \log_{10} 16$

Now, $2 \log_{10} (x + 1) = \log_{10} 16$ or, $\log_{10} (x + 1)^2 = \log_{10} 16$ or, $(x + 1)^2 = 16 = (\pm 4)^2$

or, $x + 1 = \pm 4$

$\therefore$ $x = 3, -5$.

Answer 8. (b)

L. H. S. $= \log_{2a} a.\ \log_{3a} 2a.\ \log_{4a} 3a + 1$

$= (\log_{10} a \times \log_{2a} 10) \cdot (\log_{10} 2a \times \log_{3a} 10) \cdot (\log_{10} 3a \times \log_{4a} 10) + 1$

$= \dfrac{\log_{10} a}{\log_{10} 2a} \times \dfrac{\log_{10} 2a}{\log_{10} 3a} \times \dfrac{\log_{10} 3a}{\log_{10} 4a} + 1$

$= \dfrac{\log_{10} a}{\log_{10} 4a} + 1 = \log_{4a} a + \log_{4a} 4a = \log_{4a} (a \cdot 4a) = \log_{4a} 4a^2$.

R.H.S. $= 2\log_{3a} 2a \cdot \log_{4a} 3a = \log_{4a} (2a)^2 = \log_{4a} \cdot 4a^2$

Hence the result.

Answer 8. (c)

Let, $x = \sqrt{3\sqrt{3\sqrt{3}}} \ldots$ or $x^2 = 3\sqrt{3\sqrt{3\ldots\infty}}$

(squaring both sides)

or, $x^2 = 3x$ or, $x^2 - 3x = 0$ or, $x (x - 3) = 0$ or, $x - 3 = 0$ (as $x \neq 0$),

$\therefore$ $x = 3$

$\therefore$ given expression $= \log_3 3 = 1$.

Q. 9. (a) In a class of 30 students, 15 students have taken Hindi, 10 students have taken Hindi but not English. All the students in the class have taken at least one of the subjects of English and Hindi. Find the number of students who have taken English but not Hindi.

(b) A student is to answer 8 out of 10 questions on an examination :

(i) How many choice has he?

(ii) How many if he must answer the first three questions?

(iii) How many if he must answer at least four of the first five questions?

Answer 9. (a)

$n(E \cup H) = 30, \quad n(H) = 15 \quad n(E' \cap H) = 10$

$n(E \cap H) = 15 - 10 = 5$

$n(E \cup H) = n(E) + n(H) - n(E \cap H) \Rightarrow n(E) = 30 - (15 - 5) = 20$

$n(E \cap H') = n(E) - n(E \cap H) = 20 - 5 = 15$

So, the no. of students who have taken English but not Hindi is 15.

Answer 9. (b)

(i) The 8 questions out of 10 questions may be answered in $^{10}C_8$

Now $^{10}C_8 = \dfrac{10!}{8!\,2!} = \dfrac{10 \times 9 \times (8)!}{8!\,2!} = 5 \times 9 = 45$ ways

(ii) The first 3 questions are to be answered. So there are remaining 5 (= 8 – 3) questions to be answered out of remaining 7 (= 10 – 3) questions which may be selected in $^7C_5$ ways.

Now, $^7C_5 = 7.6 = 42$ ways.

(iii) Here we have the following possible cases :

(a) 4 questions from first 5 questions (say, group A), then remaining 4 questions from the balance of 5 questions (say, group B).

(b) Again 5 questions from group A, and 3 questions from group B.

For (a), number of choice is $^5C_4 \times {}^5C_4 = 5 \times 5 = 25$

For (b), number of ways is $^5C_5 \times {}^5C_3 = 1 \times 10 = 10.$

Hence, Required no. of ways = 25 + 10 = 35.

Q. 10. (a) If $x \propto y$, prove that $px + qy \propto ax + by$, where p, q, a, b are fixed constants.

(b) If $x + y \propto x - y$, show that $ax + by \propto px + qy$, a, b, p, q being all constants.

(c) Find x, if $x^{x\sqrt{x}} = (x\sqrt{x})^x$.

Answer 10. (a)

Since $x \propto y$, we have $x = ky$, where k is a constant.

Now $\dfrac{px + qy}{ax + by} = \dfrac{p \cdot ky + qy}{a \cdot ky + by} = \dfrac{y(pk + q)}{y(ak + b)} = \dfrac{pk + q}{ak + b} =$ constant = k′ (say),

or, $px + qy = k'(ax + by)$, where k′ is a constant.

Hence $px + qy \propto ax + by$.

Answer 10. (b)

If $x + y \propto x - y$; $\therefore$ $x + y = m(x - y)$, where m is a constant,

or, $x + y = mx - my$,

or, $y + my = mx - x$,

or, $y(1 + m) = (m - 1)x$,

or, $y = \frac{m-1}{m+1}x = kx$, where k is a constant. Now see 10(a).

Answer 10. (c)

$x^{x\sqrt{x}} = (x\sqrt{x})^x$,

or, $x^{x.x^{1/2}} = (x \cdot x^{\frac{1}{2}})^x$,

or, $x^{x^{3/2}} = (x^{3/2})^x$,

or, $x^{x^{3/2}} = x^{3x/2}$;

$\therefore$ $x^{3/2} = \frac{3x}{2}$,

or, $x^3 = \frac{9}{4}x^2$,

or, $x = \frac{9}{4}$ [$\because$ $x \neq 0$]

Section - III

[Mensuration]

Q. 11. (a) The length, breadth and height of a box are 12m, 4 m and 3 m respectively. The length of the largest rod that can be placed in the box is

(i) 15m (ii) 13m (iii) 12m (iv) none of these

(b) If the hypotenuse of a right angled isosceles triangle is 4 cm then the area of the triangle is

(i) 12 sq. cm (ii) 8 sq. cm (iii) 4 sq. cm (iv) none of these

Answer 11. (a)

(ii) Length of the largest rod = $\sqrt{12^2 + 4^2 + 3^2}$ = 13m

Answer 11. (b)

(iii) Let the lendth of equal sides be d Cm.

$\therefore$ Length of hypotenuse = $\sqrt{d^2 + d^2} = d\sqrt{2}$ cm

Now $d\sqrt{2} = 4 \Rightarrow d = 2\sqrt{2}$ cm

Area of the triangle $= \frac{1}{2}d^2 = \frac{1}{2}(2\sqrt{2})^2$ = 4 sq. cm.

Q. 12. (a) The height of the right circular cone is 42 cm and its slant height is 45.5 cm. Find the cost of painting of its total surface at the rate of ₹ 1 per sq cm. (Take π = 22/7).

(b) A right pyramid with height 8 cm stands on a base which is a triangle with sides of lengths 3 cm, 4 cm and 5 cm. Find the volume of the pyramid.

Answer 12. (a)

Here $r = \sqrt{l^2 - h^2} = \sqrt{2070.25 - 1764} = \sqrt{306.25}$ = 17.5 cm

Total surface area $= \pi rl + \pi r^2 = \frac{22}{7} \times 17.5(45.5 + 17.5)$ = 3465 sq. cm.

So total cost comes to ₹ 3465

Answer 12. (b)

Semi perimeter of base = S = $\frac{3+4+5}{2}$ = 6 cm

Area of base $\sqrt{6(6-3)(6-4)(6-5)} = \sqrt{6 \times 3 \times 2 \times 1}$ = 6 sq. cm

Volume of the pyramid = $\frac{1}{3}$ X Area of base height

= $\frac{1}{3} \times 6 \times 8$ = 16 cu. cm

Q. 13. (a) The perimeter of a rectangle, having area 18 sq. cm and its length being twice its breadth, is (i) 9 cm (ii) 18 cm, (iii) 24 cm, (iv) none of these

(b) Find the quantity of water in litre flowing out of a pipe of cross-section area 5 cm^2 in 1 minute if the speed of the water in the pipe is 30 cm/sec.

(c) The volumes of two spheres are in the ratio 8:27 and the difference of their radii is 3 cm. Find the radii of both the spheres.

Answer 13. (a)

(ii) Given l = 2b and lb = 18 $\Rightarrow$ 2b^2 = 18 $\Rightarrow$ b = 3 cm

So l = 6 cm. Perimeter = 2 (1+b) = 18 cm.

Answer 13. (b)

Volume of water flowing in 1 sec = 5 × 30 = 150 c.c.

Volume of water flowing in 1 min = 150 × 60 = 9000 c.c. = 9 litre

Answer 13. (c)

$$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{8}{27} \Rightarrow \frac{r_1}{r_2} = \frac{2}{3} \Rightarrow r_1 = 2k,\ r_2 = 3k$$

Now $3k - 2k = 3 \Rightarrow k = 3$

So, the radii (r_1) of 1st sphere = 6 cm and
the radii (r_2) of 2nd sphere = 9 cm.

Q. 14. (a) The circumference of the base of a cylinder is 44 cms. and its height is 20 cms. Find the volume of the cylinder.

(b) A solid cylindrical rod of length 80 cms. radius, 15 cms. is melted and made into a cube. Find the side of the cube.

Answer 14. (a)

$2\pi r = 44$ or, $2 \times \frac{22}{7} \times r = 44$ or, $r = 7$ cm.

Volume = $\pi r^2 h = \frac{22}{7} \times 49 \times 20 = 3080$ cu. cm.

Answer 14. (b)

Volume of cube = a^3 cu. cm. [Where a is the side of cube]

Volume of cylindrical rod $= \pi r^2 h = \frac{22}{7} \times 15^2 \times 80 = a^3$, by question.

Or, $a^3 = 56571.428$ or, $a = \sqrt[3]{56571.428}$ cm.

Q. 15. (a) The circumference of the base of a cylinder is 44 cm and its height is 20 cm. Find the volume of the cylinder.

(b) The curved surface of a cylinder is 1000 sq cm and the diameter of the base is 20 cm. Find the volume of the cylinder and its height to the nearest millimeter.

Answer 15. (a)

If r cm be the radius of the base of the cylinder, then

$$2\pi r = 44, \text{ or, } r = \frac{44}{2\pi} = \frac{44}{2\pi} = \frac{44}{2} \times \frac{7}{22} = 7.$$

Volume of the cylinder $= \pi r^2 \cdot h = \frac{22}{7} \times 7^2 \times 20 = 3080$ cu cms.

Answer 15. (b)

If r cm be the radius of the base of the cylinder, then $r = \frac{20}{2} = 10$; curved surface = $2\pi rh$.

$\therefore$ $2\pi rh = 1000$, or, $\pi h = 50$, where h = height.

$\therefore$ Volume of the cylinder = $\pi r^2 h = \pi \cdot 10^2 \cdot h = 100 \times 50 = 5000$ cu cm.

Height of the cylinder $= h = \frac{50}{\pi} = 50 \times \frac{7}{22} = \frac{175}{11} = 15.909 = 15.9$ cm.

Q. 16. (a) The diameter of the base of a conical water tank is 28 m and its height is 18 m. How much water does the tank hold?

(b) A conical tent is required to accommodate 5 people, each person must have 16 sq ft of space on the ground and 100 cu ft of air to breathe. Give the vertical height, slant height and width of the tent.

Answer 16. (a)

Radius of the circular base of the tank $= \frac{28}{2} = 14$ m and its height = 18 m.

Volume of the water tank = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \cdot (14)^2 \cdot 18 = \frac{22}{7} \times 196 \times 60$

= 3696 cu m. = 3696 × 1000 decimetres

= 3696000 litres = 3696 kilolitres.

Answer 16. (b)

Space required to accommodate 5 people = 16 × 5 = 80 sq ft; $\therefore$ $\pi r^2 = 80$, where r is the radius of the base.

Again volume of the conical tent $= \frac{1}{3}\pi r^2 h$, where h = height of the tent

$= \frac{1}{3} \times 80 \times h$.

$\therefore$ $\frac{1}{3} \times 80 \times h = 100 \times 5$, or, $h = \frac{3 \times 500}{80} = 1875$ ft.

From $\pi r^2 = 80$, we have

$\frac{22}{7} r^2 = 80$, or, $r^2 = \frac{560}{22} = 25.45$; $\therefore$ r = 5.04 ft.

$\therefore$ Width = 2r = 2 × 5.04 = 10.08 ft.

l = slant height $= \sqrt{h^2 + r^2} = \sqrt{(18.75)^2 + 25.45} = \sqrt{377.01} =$ 19.42 ft.

Section - IV

[Co-ordinate Geometry]

Q. 17. (a) If the point (x, 0) is equidistant from the points (-1, 3) and (6, 4) then value of x is

(i) 1 (ii) 3 (iii) 4 (iv) none of these

(b) In what ratio is the joint of the points (4, – 1) and (5, 3) divided by the line x + 3y – 8 = 0?

Answer 17. (a)

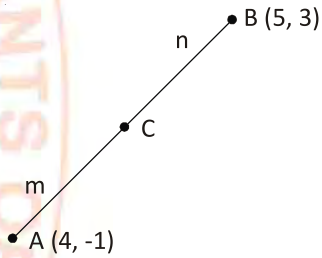
(ii) Distance between (x, 0) and (-1, 3) = Distance between (x, 0) and (6, 4)

$$\therefore \sqrt{(x+1)^2+(0-3)^2} = \sqrt{(x-6)^2+(0-4)^2}$$

$$\Rightarrow x^2+2x+10 = x^2-12x+52 \Rightarrow 14x = 42 \Rightarrow x = 3$$

Answer 17. (b)

Let the line x + 3y – 8 = 0 divide the join of A (4, – 1) and B (5, 3) at the point C in the ratio m : n. Then the co-ordinates of C are C $\left(\frac{5m+4n}{m+n}, \frac{3m-n}{m+n}\right)$.



Since C lies on the line x + 3y – 8 = 0;

$$\therefore \frac{5m+4n}{m+n} + 3\cdot\frac{3m-n}{m+n} - 8 = 0, \text{ or, } \frac{5m+4n+9m-3n-8m-8n}{m+n} = 0,$$

or, 6m – 7n = 0, or, 6m = 7n, or, $\frac{m}{n} = \frac{7}{6}$, i.e., m : n = 7 : 6.

Hence the required ratio is 7 : 6.

Q. 18. (a) Prove that the two circles $x^2 + y^2 + 2x - 6y + 5 = 0$ and $x^2 + y^2 + 10x - 2y + 21 = 0$ touch each other externally.

(b) Show that the point (3, 7) lies inside the circle $x^2 + y^2 - 6x - 8y - 11 = 0$.

Answer 18. (a)

$x^2 + y^2 + 2x - 6y + 5 = 0.$

$\Rightarrow$ > $[x + 1]^2 + (y-3)^2 = 5$ $\therefore$ Centre is (-1, 3), Radius = $r_1 = \sqrt{5}$

$x^2 + y^2 + 10x - 2y + 21 = 0$

= > $(x+5)^2 + (y-1)^2 = 5.$ $\therefore$ Centre is (-5, 1), Radius = $r_2 = \sqrt{5}$

Distance between the two centres

$$= \sqrt{(-1+5)^2+(3-1)^2} = 2\sqrt{5}$$

Again $r_1 + r_2 = \sqrt{5} + \sqrt{5} = 2\sqrt{5}$

$\therefore$ The circle touch each other externally.

Answer 18. (b)

Here $2g = -6$, $2f = -8$ and $c = -11$. $\therefore$ $g = -3$, $f = -4$.

$\therefore$ The centre C of the circle is the pont $(-g, -f) = (3, 4)$, and the radius of the circle

$$= \sqrt{g^2 + f^2 - c} = \sqrt{9 + 16 + 11} = 6.$$

The distance of the point P (3, 7) from the centre

$$C\,(3, 4) = \sqrt{(3-3)^2 + (7-4)^2} = 3,$$

which is less than 6.

Hence the point (3, 7) lies inside the circle.

Q. 19. (a) Find the co-ordinates of the vertex and the focus and the equation of the directrix of the parabola $3y^2 = 16x$. Find also the length of the latus rectum.

(b) The major and minor axes of an ellipse are the x and y axes respectively. Its eccenricity is $1/\sqrt{2}$ and the length of the latus rectum is 3 units. Find the equation of the ellipse.

Answer 19. (a)

We have

$$3y^2 = 16x, \text{ or, } y^2 = \frac{16}{3}x$$

which is of the form $y^2 = 4ax$.

Here

$$4a = \frac{16}{3}, \text{ or, } a = \frac{4}{3}.$$

$\therefore$ The co-ordinates of the vertex are (0, 0) and the co-ordinates of the focus are (a, 0), i.e., (4/3, 0).

The equation of the directrix is

$$x + a = 0, \text{ or, } x + \frac{4}{3} = 0, \text{ or, } 3x + 4 = 0.$$

The length of the latus rectum 4a = 16/3 units.

Answer 19. (b)

Equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Here, $\frac{2b^2}{a} = 3$ .... (i)

$e^2 = \frac{1}{2} \Rightarrow 1 - \frac{b^2}{a^2} = \frac{1}{2} \Rightarrow \frac{b^2}{a^2} = \frac{1}{2}$ .... (ii)

From (i) and (ii), $\frac{b^2}{a^2} = \frac{3}{2a} = \frac{1}{2} \Rightarrow a = 3$

and $b^2 = \frac{9}{2}$

Equation of the ellipse $\frac{x^2}{9} + \frac{y^2}{9/2} = 1 \Rightarrow x^2 + 2y^2 = 9$

Q. 20. (a) Find the co-ordinates of the foci, the eccentricity and the equations of the directrices of the hyperbola $16x^2 - 9y^2 = 144$.

(b) Show that the line $x - 3y = 13$ touches the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$. What are the co-ordinates of the point of contact?

Answer 20. (a)

We have

$$16x^2 - 9y^2 = 144, \text{ or, } \frac{16x^2}{144} - \frac{9y^2}{144} = 1, \text{ or, } \frac{x^2}{9} - \frac{y^2}{16} = 1.$$

Here $a^2 = 9$ and $b^2 = 16$. $\therefore$ $a = 3$, taking positive sign.

Now

$$e^2 = \frac{a^2 + b^2}{a^2} = \frac{9+16}{9} = \frac{25}{9}; \quad \therefore \ e = \frac{5}{3} \quad [\because \ e > 1.]$$

$\therefore$ The co-ordinates of the foci are

$$(\pm ae, 0) = (\pm 3.\tfrac{5}{3}, 0) = (\pm 5, 0).$$

The eccentricity is $e = \frac{5}{3}$.

The equations of the directrices are

$$ex \pm a = 0, \text{ or, } \frac{5}{3}x \pm 3 = 0 \text{ or, } 5x \pm 9 = 0.$$

Answer 20. (b)

$$x - 3y = 13 \qquad \text{(i)}$$

$$\text{and } \frac{x^2}{25} + \frac{y^2}{16} = 1.$$

From (1),

$$x = 13 + 3y.$$

$\therefore$ From (2),

$$\frac{(13+3y)^2}{25} + \frac{y^2}{16} = 1, \text{ or } \frac{169 + 9y^2 + 78y}{25} = \frac{y^2}{16} = 1, \text{ or, } \frac{2704 + 144y^2 + 1248y + 25y^2}{400} = 1,$$

$$\text{or, } 169y^2 + 1248y + 2304 = 0, \text{ or, } (13y + 48)^2 = 0$$

which gives two real and equal values of y, i.e., $y = -\frac{48}{18}, -\frac{48}{18}$.

Since the roots are equal, the line (i) intersects the ellipse (ii) in two coincident points. Hence the line (i) touches the ellipse (ii).

Substituting $y = \frac{-48}{13}$ in $x = 13 + 3y$, we get

$$x = 13 + 3 \times \frac{-48}{13} = \frac{169 - 144}{13} = \frac{25}{13}.$$

$\therefore$ The point of contact is $\left(\frac{25}{13}, \frac{-48}{13}\right)$.

Section - V
[Calculus]

Q. 21. (a) If $y = \log\left(x+\sqrt{x^2+a^2}\right)$ then prove that $(a^2+x^2)\,y_2+xy_1=0$.

(b) If $y = Ae^{mx} + Be^{-mx}$ show that $y_2 - m^2y = 0$.

(c) Find the area of the region bounded by curves $y^2 = x$ and $y = x$.

Answer 21. (a)

$$y = \log\left(x+\sqrt{x^2+a^2}\right)$$

$$y_1 = \frac{1+\frac{1}{2}(x^2+a^2)^{-\frac{1}{2}}.2x}{x+\sqrt{x^2+a^2}} = \frac{1+\frac{x}{\sqrt{x^2+a^2}}}{x+\sqrt{x^2+a^2}} = \frac{1}{\sqrt{x^2+a^2}}$$

$$y_2 = -\frac{1}{2}(x^2+a^2)^{-\frac{3}{2}}.2x = -\frac{x}{(x^2+a^2)\sqrt{x^2+a^2}}$$

$$\therefore\ \left(x^2+a^2\right)y_2+xy_1=0$$

Answer 21. (b)

$y \ = Ae^{mx} + Be^{-mx}$

$y_1 \ = Ame^{mx} - Bme^{-mx}$

$\ \ = m\,(Ae^{mx} - Be^{-mx})$

$y_2 \ = m\,(Ame^{mx} + Bme^{-mx})$

$\ \ = m^2\,(Ae^{mx} + Be^{-mx}) = m^2y$

$\therefore\ y_2 - m^2y = 0$

Answer 21. (c)

$y^2 = x$ and $y = x$ cut at (0, 0) and (1, 1)

$$\text{Required area} = \int_0^1 \sqrt{x}\,dx - \int_0^1 x\,dx$$

$$\left[\frac{x^{2/3}}{3/2}\right]_0^1 - \left[\frac{x^2}{2}\right]_0^1 = \frac{2}{3}(1-0) - \frac{1}{20}(1-0) = \frac{1}{6} \text{ sq. unit}$$

Q. 22. (a) If $x^a y^b = (x+y)^{a+b}$ show that $\frac{dy}{dx} = \frac{y}{x}$ where *a* and *b* are independent of *x* and *y*.

(b) If $y = \left(x + \sqrt{1+x^2}\right)^m$ show that $(1+x^2)y_2 + xy_1 = m^2y$.

(c) The value of $\int_0^1 \frac{dx}{x+\sqrt{x}}$ is

(i) $\log_e 2$, (ii) $2\log_e 2$, (iii) $-\log_e 2$, (iv) none of these

Answer 22. (a)

a log x + b log y = (a+b) log (x+y)

Differentiating w.r.t. to x.

$$\frac{a}{x} + \frac{b}{y} \cdot \frac{dy}{dx} = \frac{a+b}{x+y}\left(1 + \frac{dy}{dx}\right)$$

or, $$\left(\frac{b}{y} - \frac{a+b}{x+y}\right)\frac{dy}{dx} = \frac{a+b}{x+y} - \frac{a}{x}$$

or, $$\frac{b(x+y)-(a+b)y}{y(x+y)} \cdot \frac{dy}{dx} = \frac{x(a+b)-a(x+y)}{x(x+y)}$$

or, $$\frac{dy}{dx} = \frac{bx-ay}{x(x+y)} \times \frac{y(x+y)}{(bx-ay)} = \frac{y}{x}$$

Answer 22. (b)

$$y = \left(x+\sqrt{1+x^2}\right)^m \Rightarrow y_1 m\left(x+\sqrt{1+x^2}\right)^{m-1}\left\{1+\frac{1\times 2x}{2\sqrt{1+x^2}}\right\}$$

$$\Rightarrow y_1 = m\left(x+\sqrt{1+x^2}\right)^{m-1}\left(\frac{x+\sqrt{1+x^2}}{\sqrt{1+x^2}}\right)$$

$$\Rightarrow y_1 = \frac{my}{\sqrt{1+x^2}} \Rightarrow y_1^2(1+x^2) = m^2y^2$$

$$\therefore\ 2y_1y_2\,(1+x^2) + 2xy_1^2 = 2m^2yy_1$$

$$\Rightarrow y_2(1+x^2) + xy_1 = m^2y$$

$(1+x^2)\,y_2 + xy_1 = m^2y$ Proved.

Answer 22. (c)

(ii) $I = \int_0^1 \frac{dx}{x+\sqrt{x}} = \int_0^1 \frac{dx}{\sqrt{x}+\left(\sqrt{x}+1\right)} = \int_1^2 \frac{2dy}{y}$ where $\sqrt{x}+1=y$ i.e. $\frac{dx}{2\sqrt{x}}=dy$

So $I = \left[2\log_e y\right]_1^2 = 2\log e^2$

Q. 23. (a) If $y=\log\left(x+\sqrt{x^2+a^2}\right)$ $y_2 + xy_1 = 0$.

(b) If sum of two values is 8 find the maximum value of their product.

(c) Find the area of the region bounded by curves $y^2 = x$ and $y = x$.

(d) If $u= x^2+y^2+z^2$, the value of $xu_x + yu_y + zu_z$ is

(i) 2u (ii) 2 (iii) -2u (iv) none of these

Answer 23. (a)

$y = \log\left(x+\sqrt{x^2+a^2}\right)$

$$y_1 = \frac{1+\frac{1}{2}(x^2+a^2)^{-\frac{1}{2}}.2x}{x+\sqrt{x^2+a^2}} = \frac{1+\frac{x}{\sqrt{x^2+a^2}}}{x+\sqrt{x^2+a^2}} = \frac{1}{\sqrt{x^2+a^2}}$$

$$y_2 = -\frac{1}{2}(x^2+a^2)^{-\frac{3}{2}.2x} = -\frac{x}{(x^2+a^2)\sqrt{x^2+a^2}}$$

$$\therefore\ (x^2+a^2)y_2 + xy_1 = 0$$

Answer 23. (b)

Let two values be x and y

Then $x + y = 8$. Let $A = xy = x(8 - x) = 8x - x^2$

$\frac{dA}{dx} = 8 - 2x.$ $\frac{dA}{dx} = 0$ give $x = 4$

$\frac{d^2A}{dx^2} = -2 < 0$. So A is maximum at $x = 4$

$x = 4$, then $y = 8 - 4 = 4$

$\therefore$ $xy = 4.4 = 4^2 = 16$.

So, maximum $A = 84 - 4^2 = 16$.

Answer 23. (c)

$y^2 = x$ and $y = x$ cut at (0, 0) and (1, 1)

Required area $= \int_0^1 \sqrt{x}dx - \int_0^1 xdx$

$$\left[\frac{x^{3/2}}{3/2}\right]_0^1 - \left[\frac{x^2}{2}\right]_0^1 = \frac{2}{3}(1-0) - \frac{1}{2}(1-0) = \frac{1}{6} \text{ sq. unit}$$

Answer 23. (d)

(i)

$u = x^2 + y^2 + z^2$

$u_x = 2x$

$u_y = 2y$

$u_z = 2z.$

$\therefore\ xu_x + yu_y + zu_z = x(2x) + y(2y) + z(2z) = 2(x^2+y^2+z^2) = 2u$

Q. 24. (a) The total cost (C) for output x is as follows :

$$C = \frac{3}{5}x + \frac{15}{4}.$$

Find (i) Cost when output is 5 units (ii) Average cost of output of 10 units (iii) Marginal cost (C)

(b) The cost function (C) for commodity (q) is given by $C = q^3 - 4q^2 + 6q$. Find the AVC and also find the value of q for which AVC is minimum.

(c) If $y = x^2 \log_e x$, show that $x^2 \frac{d^2y}{dx^2} + 4y = 3x\frac{dy}{dx}$

Answer 24. (a)

For 5 units $= \frac{3}{5}.5 + \frac{15}{4} = 3 + 3.75 = 6.75$ units.

$$AC = \frac{C}{x} = \frac{\frac{3}{3}x + \frac{15}{4}}{x} = \frac{3}{5} + \frac{15}{4x}$$

$\therefore$ AC for 10 units $= \frac{3}{5} + \frac{15}{40} = 0.6 + 0.375 = 0.975;$ $MC = \frac{d(C)}{dx} = \frac{3}{5} = 0.6.$

Answer 24. (b)

$AVC = \frac{C}{q} = q^2 - 4q + 6,$ (in cost function (C), fixed cost is absent)

$MC = \frac{d}{dq}(q^3 - 4q^2 + 6q) = 3q^2 - 8q + 6.$ For AVC minimum, slope of AC is zero i.e.,

$\frac{d}{dq}(q^2 - 4q + 6) = 0$ or, $2q - 4 = 0$ or, $q = 2$ units.

Answer 24. (c)

$y = x^2 \log_e x$

$\frac{dy}{dx} = x^2 \cdot \frac{1}{x} + 2x \cdot \log_e x$

$= x + 2x\log_e x$

$\frac{d^2y}{dx^2} = 1 + \left(2x \cdot \frac{1}{x} + 2\log_e x\right)$

$= 3 + 2\log_e x$

$x^2 \frac{d^2y}{dx^2} + 4y = 3x^2 + 2x^2\log_e x + 4x^2\log_e x = 3x(x + 2x\log_e x) = 3x.\frac{dy}{dx}$

Section - VI
[Statistics]

Q. 25. (a) Draw a Pie Chart to represent the following data relating to the production cost of manufacture : Cost of Materiala—₹ 38,400, Cost of Labour—₹ 30,720; Direct Expenses of Manufacture—₹ 11,520; Factory Overhead expenses—₹ 15,360

(b) From the following distribution of wages of workers in a factory, draw a histogram and a frequency polygon—

Wages (in ₹) :	**0-10**	**10-20**	**20-40**	**40-50**	**50-80**	**80-100**
No. of Workers :	**10**	**20**	**50**	**15**	**12**	**12**

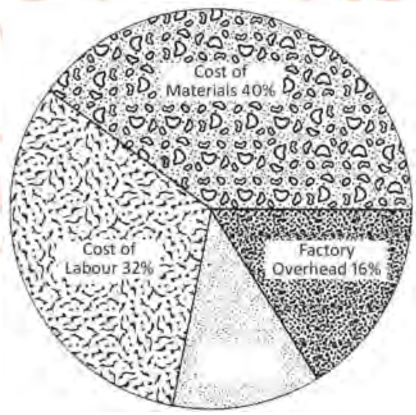
Answer 25. (a)

We first express each item as a percentage of the total cost, viz., Rs. 96,000.

	Percentage	**Central angles**
1. Cost of Materials	$\frac{38,400}{96,000} \times 100\% = 40\%$	$3°.6 \times 40 = 144°$
2. Cost of Labour	$\frac{30,720}{96,000} \times 100\% = 32\%$	$3°.6 \times 32 = 115°.2$
3. Direct Expenses	$\frac{11,520}{96,000} \times 100\% = 12\%$	$3°.6 \times 12 = 43°.2$
4. Factory Overhead	$\frac{15,360}{96,000} \times 100\% = 16\%$	$3°.6 \times 16 = 57°.6$
Total	= 100%	= 360°

A circle of conveient redius is now drawn and the above angles are marked out at the centre of the circle. 4 radii will then divide the whole circle into four required sectors.

The different sectors are generally differently shaded. fig. gives the Pie Chart required.



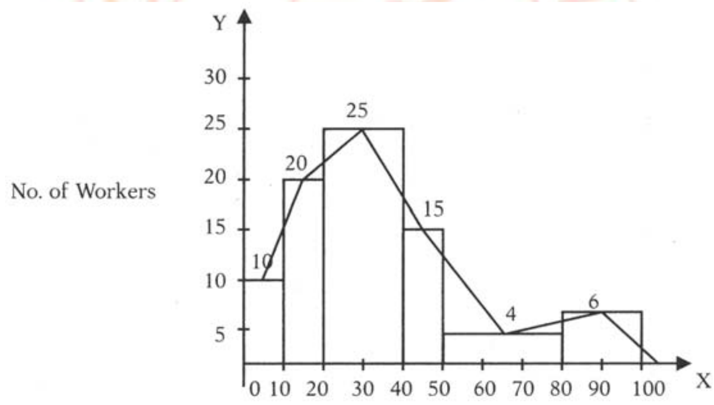
The calculations of angles at the centre can be avoided if the circumstance of the circle be divided into 100 equal parts.

Answer 25. (b)

As the class internal are unequal, we have to find frequency density so that height of the rectangle could be calculated.

(1) Class interval	(2) Class width	(3) frequency	(4) = (3) ÷ (2) frequency density	(5) = (4) × 10 height of the rectangle
0 - 10	10	10	1	10
10 - 20	10	20	2	20
20 - 40	20	50	10	25
40 - 50	10	15	2.5	15
50 - 80	30	12	1.5	4
80 - 100	20	12	0.4	6

Histogram and Frequency Polygon



Wages (in ₹)

Q. 26. (a) The mean annual salary of all employees of a company is Rs. 28,500. The mean salaries of male and female employees are Rs. 30,000 and Rs. 25,000, respectively. Find the percentage of males and females employed by the company.

(b) Median marks of 50 candidates in mathematics in a test are 26. Frequencies in the ranges 10 - 20 and 30 - 40 are missing the following table :

Marks obtained :	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**
No. of candidates :	**5**	**—**	**20**	**—**	**7**

Determine the missing frequencies.

Answer 26. (a)

Let n_1 be the number of males and n_2 the number of females

Then $x = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

$$\Rightarrow 28{,}500 = \frac{n_1 \times 30{,}000 + n_2 \times 25{,}000}{n_1 + n_2}$$

$\Rightarrow 3{,}500n_2 = 1500n_i$

$\Rightarrow \dfrac{n_1}{n_2} = \dfrac{70}{30} \Rightarrow n_1 : n_2 = 7:3$

$\therefore$ Percentage of males $= \dfrac{7}{10} \times 100 = 70$ and females = 30.

Answer 26. (b)

Let the missing frequencies be f_1 and f_2

$\therefore$ $5 + f_1 + 20 + f_2 + 7 = 50 \Rightarrow f_1 + f_2 = 18$

Class (Marks)	No. of Candidates	Cumulative frequencies
0 - 10	5	5
10 - 20	f_1	$5 + f_1$
20 - 30	20	$25 + f_1$ * median class
30 - 40	f_2	$25 + f_1 + f_2$
40 - 50	7	$32 + f_1 + f_2$

Median $= 26 = 20 + \dfrac{25 - (5 + f_1)}{20} \times 10,$

$\Rightarrow 6 \times 20 = 200 - 10f_1 \Rightarrow f_1 = 8$

$f_1 + f_2 = 18 \Rightarrow f_2 = 18 - 8 = 10$

$\therefore$ The missing frequencies are 8 and 10.

Q. 27. (a) Find the mean deviation about arithmetic mean of the first 10 natural numbers.

(b) Following are the marks obtained by 50 students in mercantile law paper :

Marks	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**
Students	**4**	**6**	**20**	**10**	**7**	**3**

Find median graphically.

Answer 27. (a)

Sum of first 10 natural numbers $= \dfrac{n(n+1)}{2} = \dfrac{10 \times 11}{2} = 55$

A.M. of first 10 natural numbers $= \dfrac{55}{10} = 5.5$

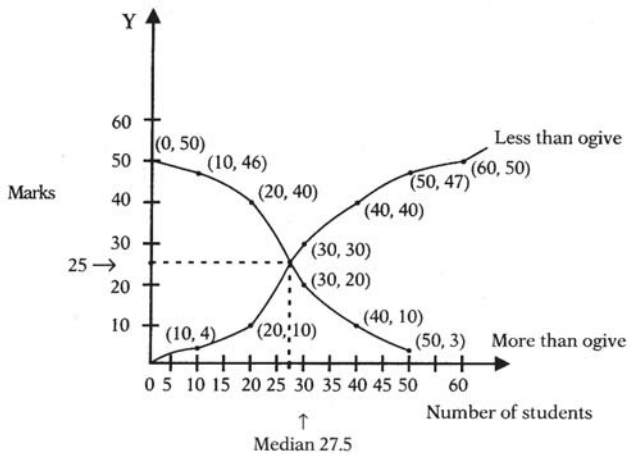
First 10 natural numbers (x)	$x-\bar{x}$ $= x-5.5$	$\lvert x-5.5 \rvert$	
1	−4.5	4.5	
2	−3.5	3.5	Mean deviation
3	−2.5	2.5	
4	−1.5	1.5	$=\frac{25}{10}=2.5$
5	−0.5	0.5	
6	0.5	0.5	
7	1.5	1.5	
8	2.5	2.5	
9	3.5	3.5	
10	4.5	4.5	
		25	

Answer 27. (b)

Marks	Freq	Marks more than		Marks less than	
0 - 10	4	0	50	10	4
10 - 20	6	10	50 - 4 = 46	20	4 + 6 = 10
20 - 30	20	20	46 - 6 = 40	30	10 + 20 = 30
30 - 40	10	30	40 - 20 = 20	40	30 + 10 = 40
40 - 50	7	40	20 - 10 = 10	50	40 + 7 = 47
50 - 60	3	50	10 - 7 = 3	60	47 + 3 = 50

Median could be found graphically by making more than ogive and less than ogive.
Where ever these two ogives meet, draw perpenditular from that point to X-axis to get the value of median.

Computation of Median graphically



Q. 28. (a) Find the mean and standard deviation of the 2 values, (a + b) and (a – b).

(b) Prove that for any two positive real quantities AM $\geq$ GM $\geq$ HM.

(c) Calculate coefficient of variation for the following distribution of marks obtained by 60 students in a test :

Marks	**:**	**0 — 10**	**10 — 20**	**20 — 30**	**30 — 40**	**40 — 50**
Students	**:**	**11**	**19**	**15**	**9**	**6**

Answer 28. (a)

$$\text{Mean} = \frac{a+b+a-b}{2} = a.$$

$$\text{S.D} = \sqrt{\frac{\Sigma\,(x-\bar{x})^2}{n}} = \sqrt{\frac{(a+b-a)^2}{2} + \frac{(a-b-a)^2}{2}} = \sqrt{\frac{b^2}{2} + \frac{b^2}{2}} = b\,.$$

Answer 28. (b)

Let x_1 and x_2 be any two positive real quantities.

Now $(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2$

$\Rightarrow (x_1 + x_2)^2 - 4x_1x_2 \geq 0$

$\Rightarrow \left(\frac{x_1+x_2}{2}\right) \geq x_1x_2 \Rightarrow \frac{x_1+x_2}{2} \geq \sqrt{x_1x_2} \Rightarrow AM \geq GM$ ... (I)

Next $\dfrac{\frac{x_1+x_2}{x_1x_2}}{2} \geq \dfrac{\sqrt{x_1x_2}}{x_1x_2} \Rightarrow \dfrac{\frac{1}{x_1}+\frac{1}{x_2}}{2} \geq \dfrac{1}{\sqrt{x_1x_2}}$

$\Rightarrow \dfrac{2}{\frac{1}{x_1}+\frac{1}{x_2}} \leq \sqrt{x_1x_2} \Rightarrow HM \leq GM$ ... (II)

Combining (I) & (II)

$AM \geq GM \geq HM$

Answer 28. (c)

Class Interval	Mid Value *x*	No. of Students *f*	x – 25 = d	$d' = \frac{d}{10}$	fd′	fd'^2
0 — 10	5	11	– 20	– 2	– 22	44
10 — 20	15	19	– 10	– 1	– 19	19
20 — 30	25	15	0	0	0	0
30 — 40	35	9	10	1	9	9
40 — 50	45	6	20	2	12	24
	$\Sigma f = 60$				$\Sigma fd' = -20$	$fd'^2 = 96$

Mean $= A + \left(\frac{\Sigma fd'}{\Sigma f}\right) \times i = 25 - \frac{20}{60} \times 10 = 25 - \frac{20}{6} = \frac{130}{6} = 21.7$ approx.

S.D. $= 10\sqrt{\left[\frac{\Sigma fd'^2}{\Sigma f} - \left(\frac{\Sigma fd'}{\Sigma f}\right)^2\right]}$

$= \sqrt{\frac{96}{60} - \left(\frac{-20}{60}\right)^2} = 10\sqrt{1.6 - 0.11} = 12.2$

C.V. $= \frac{\text{S.D.}}{\text{Mean}} \times 100 = \frac{12.2}{21.7} \times 100 = 56.25$

Q. 29. (a) Explain the concept is negatively skewed.

For a frequency distribution, the quartiles are ₹ 20 and ₹ 50, and the median is 30. Calculate bowley's coefficient of skewness.

(b) The means of two samples of sozes 50 and 100 respectively are 54.1 and 50.3. Obtain the mean of the sample of size 150 obtained by combining the two samples.

(c) Calculate the arithmetic mean and the median of the frequency distribution given below. Hence claculate the mode using the empirical relation between the three.

Class-limits	130-134	136-139	140-144	145-149	150-154	155-159	160-164
Frequency	5	15	28	24	17	10	1

Answer 29. (a)

Firs Quartile = 20 = Q_1

Second Quartile = Meadian = 30 = Q_2

Third Quartile = 50 = Q_3

Bowley's Coefficient of Skewness

$$= \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1} = \frac{50 + 20 - 60}{50 - 20} = \frac{10}{30} = \frac{1}{3}.$$

Answer 29. (b)

Here $n_1 = 50$, $n_2 = 100$, $\bar{x}_1 = 54.1$, $\bar{x}_2 = 50.3$.

$\therefore$ Mean $(\bar{x}) = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2} = \frac{50 \times 54.1 + 100 \times 50.3}{50 + 100} = \frac{2705 + 5030}{150} = \frac{7735}{150}$ = **51.57** (approx.)

Answer 29. (c)

Calculation of A.M.

Class-intervals	Mid-values *x*	$d=\frac{x-A}{i}$ *(i = 5)*	Frequency *f*	*fd*
130-134	132	-3	5	-15
135-139	137	-2	15	- 30
140-144	142	-1	28	- 28
145-149	147 = A	0	24	0
150-154	152	1	17	17
155-159	157	2	10	20
160-164	162	3	1	3
Total	...	...	100 = N	33 = *Efd*

Arithmetic Mean = $A+\frac{\Sigma fd}{N}\times i=147+\frac{-33}{100}\times 5=147-1.65=145.35$

CALCULATION OF CUMULATIVE FREQUENCY

Class-boundary	Cumulative Frequency (less than)
129.5	0
134.5	5
139.5	20
144.5	48
Median	x—50
149.5	72
154.5	89
159.5	99
164.5	100 = N

Here $\frac{N}{2}=\frac{100}{2}=50$. Median (M) = the value corresponding to cumulative frequency 50.

$\therefore$ Median class is 145-149 and Median $=l_1+\frac{\frac{N}{2}-C}{f}\times i$

Here l_1 = 144.5, $\frac{N}{2}$ = 50, C = 48, f = 24, i = 5.

$\therefore$ Median $=144.5+\frac{50-48}{24}\times 5=144.5+\frac{5}{12}=144.5+0.42=144.92$

The empirical relation between Mean, Median and Mode is

Mean – Mode = 3 (Mean – Median), or, 145.35 – Mode = 3 (145.35 – 144.92) = 3 × 0.43,

or, 145.35 – 1.29 = Mode, i.e. Mode = 144.06.

Q. 30. (a) (i) Find mean and standard deviation of following frequency distribution of ages :

Class of age (yrs) :	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	Total
No. of persons :	2	4	9	3	2	20

(ii) Find the median and mode of the following grouped frequency distribution :

Salaries (in ₹) per hour :	5 – 9	10 – 14	15 – 19	20 – 24	25 – 29	Total
No. of persons :	10	20	30	25	15	100

(iii) For a group containing 90 observations the mean and standard deviation are 59 and 9 respectively. For 40 observations of them mean and standard deviation are 54 and 6 respectively. Find the mean and standard deviation of the remaining 50 observations.

(b) Short notes on :

(i) Central Tendency of Data;

(ii) Ogive less than type.

Answer 30. (a)

(i)

Mid value (x) :	5	15	25	35	45	Total
u = (x – 25)/10 :	– 2	– 1	0	1	2	
freq (f) :	2	4	9	3	2	20
fu :	– 4	– 4	0	3	4	– 1
fu^2 :	8	4	0	3	8	23

$$\text{mean} = 25 + 10 \times \frac{\sum fu}{\sum f} = 25 + \frac{10 \times (-1)}{20} = 25 - 0.5 = 24.5 \text{ yrs.}$$

$$\sqrt{\left[\frac{\sum fu^2}{\sum f} - \left(\frac{\sum fu}{\sum f}\right)^2\right]} \times 10 = \sqrt{\left[\frac{23}{20} - \left(\frac{-1}{20}\right)^2\right]} \times 10 = \sqrt{114.75} = 10.71 \text{ yrs.}$$

(ii)

Class (₹)	Frequency	Cumulative frequency (< type)
4.5 - 9.5	10	10
9.5 - 14.5	20	30
14.5 - 19.5	30	60
19.5 - 24.5	25	85
24.5 - 29.5	15	100

$\frac{100}{2} = 50.$ So median class is 14.5 – 19.5 since value corresponding to 50 (C.F.) lies in that class.

$$\text{Median } = 14.5 + \frac{\frac{100}{2} - 30}{60-30} \times 5 = 14.5 + \frac{20}{30} \times 5 = 14.5 + \frac{10}{3}$$

= 14.5 + 3.33 = 17.83 ₹.

Modal class is 14.5 – 19.5 since maximum frequency 30 lies in that class

$$\text{Mode } = 14.5 + \frac{30-20}{(30-20)+(30-25)} \times 5 = 14.5 + \frac{10}{10+5} \times = 14.5 + \frac{10}{3}$$

= 14.5 + 3.33 = 17.83 ₹.

(iii) $\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

i.e., $59 = \frac{40 \times 54 + 50\bar{x}_2}{90}$ i.e., $\bar{x}_2 = 63$

$$(n_1 + n_2)\sigma^2 = n_1\left[\sigma_1^2 + (\bar{x} - \bar{x}_1)^2\right] + n_2\left[\sigma_2^2 + (\bar{x}_2 - \bar{x})^2\right]$$

$$\Rightarrow 90 \times 81 = 40\left[36 + (59-54)^2\right] + 50\left[\sigma_2^2 + (63-59)^2\right]$$

$\Rightarrow \sigma_2^2 = 81$ i.e., $\sigma_2 = 9$ = s.d of remaining 50 observations

Answer 30. (b)

(i) **CENTRAL TENDENCY OF DATA :**

A given raw statistical data can be condensed to a large extent by the methods of Classification and tabulation. But this is not enough for interpreting a given data we are to depend on some mathematical measures. Such a type of measure is the measure of Central Tendency.

By the term of Central Tendency of Data we mean that Central Value of the data about which the observations are concentrated. Since the single value has a tendency to be somewhere at the Centre and within the range of all values, it is also known as the measure of Central Tendency.

There are three measures of Central Tendency :

(i) Mean (ii) Median (iii) Mode

Mean is the most important measure which is of three types :

(i) Arithmetic mean

(ii) Geometric Mean

(iii) Harmonic Mean

Mean of a series (usually denoted by $\bar{X}$) is the value obtained by dividing the sum of the values of various items, in a series ($\sum X$) divided by the number of items (N) constituting the series.

Median : If a set of observations is arranged in order of magnitude, then the middle –most or central value gives the median. Median divides the observations into two equal parts, in such a way that the number of observations smaller than median is equal to the number greater than it.

Mode : Mode is the value of the variate which occurs with maximum frequency. It represents the most frequent value of a series.

In most frequency distributions Mean, Median and Mode obey the approximate relation known as Empirical relation expressed as Mean – Mode = 3 (Mean – Median).

(ii) **Ogive less than type :**

Cumulative frequency corresponding to a given variate value of a distribution is defined to be the sum total of frequencies up to and including that variate value. This is known as cumulative frequency of less than type. In case of a grouped frequency distribution, cumulative frequency (less than) of a class corresponds to the upper class-boundary of that class and it is the sum total of frequencies of classes up to and including that class.

For grouped frequency distribution of a continuous variable, cumulative, cumulative frequency distribution of less than type can be represented graphically by means of a cumulative frequency polygon also known as ogive less than type. To draw a cumulative frequency polygon, boundary values of each class are located in the X axis. The cumulative frequency table provides the cumulative frequency (less than type) corresponding to upper class boundary of a class along Y axis. For each pair of values (U_i, CF_i), a point is plotted in the graph paper. Joining all these points by straight lines, we get a cumulative frequency polygon of 'less than type'.